Adaptive Inflatable Structures for protecting wind turbines against ship collisions

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Abstract
Collisions of small ships are one of main dangers for the offshore wind turbines. Using inflatable structures surrounding the tower on the water level is a possibility of effective protection. Modelling of such structures is based on interaction between solid wall and fluid enclosed inside. Inflatable structures can be adapted to various impact schemes by adjusting initial pressure and controlling release of compressed air by opening piezo-valves. Simulations of ship collision with 2D model of wind turbine tower protected by pneumatic structure are presented in the report. Numerical analysis is performed using ABAQUS/Standard and ABAQUS/Explicit. Performed feasibility study proves that inflatable structures can protect wind turbine tower and ship against serious damages.

Keywords:
marine casualties, accidents, collisions, small vessels, inflatable structures, pneumatic structures, offshore wind turbines, adaptive impact absorption, modelling, ABAQUS

ISBN
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  – for adiabatic change of gas
  – for adiabatic flow through orifice
  – for dissipating impact energy
  – for mitigation of ship accelerations
### List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M, K, C</strong></td>
<td>mass, stiffness and damping matrices</td>
</tr>
<tr>
<td>k, c</td>
<td>spring stiffness and damping</td>
</tr>
<tr>
<td>q</td>
<td>vector of displacements</td>
</tr>
<tr>
<td>(V_0)</td>
<td>initial velocity of the ship</td>
</tr>
<tr>
<td>(M)</td>
<td>mass of the wind turbine, reduced mass of the wind turbine</td>
</tr>
<tr>
<td>(m)</td>
<td>mass of the ship, reduced mass of the ship</td>
</tr>
<tr>
<td>(r_1, r_2)</td>
<td>radius of the tower, outer radius of the inflatable structure</td>
</tr>
<tr>
<td>(d)</td>
<td>number of chambers in the inflatable structure</td>
</tr>
<tr>
<td>(h)</td>
<td>height of the inflatable structure</td>
</tr>
<tr>
<td>(E)</td>
<td>Young modulus of composite material constituting inflatable structure wall</td>
</tr>
<tr>
<td>(R, \bar{R})</td>
<td>gas constant (for air: 287.696 J/(kg<em>K)), universal gas const. (8.314 J/(mole</em>K))</td>
</tr>
<tr>
<td>(c_p, \bar{c}_p)</td>
<td>const. pressure heat capacity [J/(kg<em>K)], const. pressure molar heat capacity [J/(mole</em>K)]</td>
</tr>
<tr>
<td>(c_v, \bar{c}_v)</td>
<td>const. volume heat capacity [J/(kg<em>K)], const. volume molar heat capacity [J/(mole</em>K)]</td>
</tr>
<tr>
<td>(MW)</td>
<td>molecular weight of the gas (for air MW = 0.0289 kg/mole)</td>
</tr>
<tr>
<td>(q, \dot{m})</td>
<td>mass flow rate in ABAQUS / STD [kg/s] and in ABAQUS/XPL [kg/(m²*s)]</td>
</tr>
<tr>
<td>(C_V, C_H)</td>
<td>viscous flow coefficient [Pa<em>s/kg], hydrodynamic flow coefficient [Pa</em>s²/kg²]</td>
</tr>
<tr>
<td>(\theta_Z)</td>
<td>temperature of absolute zero (-273.15 C)</td>
</tr>
<tr>
<td>(p_A)</td>
<td>ambient pressure (101360 Pa)</td>
</tr>
<tr>
<td>(p_r, p_0, p)</td>
<td>reference, initial and actual gauge pressure [Pa]</td>
</tr>
</tbody>
</table>
$p_{out}$ outside gauge pressure (0 Pa)

$\bar{p}_r, \bar{p}_0, \bar{p}$ reference, initial and actual absolute (total) pressure [Pa]

$\theta_r, \theta_0, \theta$ reference, initial and actual temperature of the gas [K]

$\rho_r, \rho_0, \rho$ reference, initial and actual density of the gas [kg/m$^3$]

$W$ work done by gas [J]

$Q$ heat transferred to the system [J]

$E, \bar{E}$ internal energy [J], specific internal energy [J/kg]

$H, \bar{H}$ internal enthalpy [J], specific internal enthalpy [J/kg]
1. Introduction

The number of wind turbines is growing every year worldwide. More and more wind turbines are moved to offshore regions due to beneficial wind conditions, large spaces availability and lack of inconvenience for inhabitants. Small ships have to dock to wind turbines towers for the purpose of conservation and monitoring. Collisions with these ships during rough sea conditions are one of the biggest threats for the wind turbines. The main purpose of this report is to perform the feasibility study of inflatable structures for protecting wind turbines towers. The goal of applying pneumatic structures is mitigating of the response of both ship and wind turbine tower. In particular, inflatable structure helps to dissipate the impact energy, minimise forces acting on the ship, decrease stresses arising at the location of the collision and mitigate vibrations of the tower. The terms inflatable, pneumatic and pressurised have equivalent meaning and they will be used interchangeably.

The main numerical tool used for calculations is ABAQUS software [1] which utilises Finite Element Method (FEM) as described in [2]. ABAQUS/Standard uses implicit scheme of solution and solves equilibrium equations directly on every increment. In contrast, ABAQUS/Explicit advances the solution kinematically in small increments without solving coupled set of equations. This makes the explicit method better suited for fast dynamic or strongly nonlinear problems.

The first part of the report contains general description of Fluid Structure Interaction (FSI) modelling. Typical problems are presented together with governing differential equations. The methods for modelling fluid-filled structures in ABAQUS/Standard and ABAQUS/Explicit are presented and compared. The choice of the best methods for further calculations is done. FORTRAN subroutines for capabilities that are not available in ABAQUS are implemented. The second part deals with numerical analysis of collision between the ship and wind turbine protected by a pressurised structure. The wind turbine is reduced to a one and two-dimensional model on the water level. Parametric analysis performed on a one-dimensional model is used to investigate the basic features of the new solution. Then a two-dimensional model is applied for a more precise FEM modelling of the ship collision with inflatable structure. Various options of pneumatic structure design, as well as different strategies of pressure adjustment for alleviation of tower and ship response are considered.

The present work was completed during a half-year visit of the first author at VTT Technical Research Centre of Finland. The authors would like to gratefully acknowledge the support of VTT in accomplishing this work. Essential help provided by Tuomo Kärnä and Ilkka Hakola is especially highly appreciated.
2. The idea of Adaptive Inflatable Structures

Adaptive Inflatable Structures (AIS) are structures filled with gas under appropriately adjusted changeable pressure. They are designed for the purpose of resisting and mitigating impact forces. The idea of using compressed air and its controlled release makes pressurised structures easily adaptable for various impact forces and scenarios.

Using compressed air

Inflatable structures should be permanently inflated with a relatively low pressure providing mitigation of smaller impacts and maintaining a desired shape of the pneumatic structure. Additional deployment is planned before any stronger collisions. Pressure must be adjusted according to the velocity, mass and area of contact with the hitting object. Fast reacting pyrotechnic systems based on the concept of micro explosions (similarly as in the air bag in the car) can be used for immediate gas pumping. Pressure of the gas acting outwards increases stiffness of the pneumatic structure and prevents its huge deformation effected by the hitting object.

Release of pressure

During the process of immersing of the object into the inflatable structure controlled release of pressure is executed by opening exit piezo-valves. This way we can control the stiffness of the pneumatic structure in the subsequent stages of impact. We can stop the penetrating object in the appropriate way and prevent excessive forces and accelerations in the system. The second purpose of applying a release of pressure is to control the dissipation of the energy.

Two types of pressurised structures can be distinguished:

a) applied as a separate structure as a prototype of protective barrier, cf. Fig. 1
b) attached to an existing structure like the structure for protecting offshore wind turbine against ship collisions, cf. Fig. 2.

The first type of inflatable structure was considered in detail in paper [3] where two-dimensional thin-walled steel structure was subjected to impact load. This kind of structure is not well adapted to sustain lateral impacts. However, by using compressed air we managed to increase significantly load capacity, decrease accelerations and control structure destruction shape. Further improvement of the AIS was achieved by dividing the structure into several packages separated by flexible walls in which the piezo-valves allowing flow of the gas were mounted. In comparison to standard protective barrier, the pressurised one does not demand the use of additional to increase
its load capacity. Compressed air also has a beneficial influence on the buckling behaviour of the structure, since it increases the value of critical force. This is due to the fact that additional loading caused by the pressure acting outwards may profitably influence the distribution of the compressive forces which cause loss of the structure stability.

\[ p_1(t) \quad p_2(t) \quad p_3(t) \quad p_4(t) \quad p_5(t) \quad p_6(t) \]

Figure 1. Inflatable structure serving as a protective barrier.

A pneumatic structure attached to offshore wind turbine is considered in further part of this report. The analysed inflatable structure has the shape of torus which surrounds the tower on the water level protecting it against ship collisions. Design and dimensions of AIS are determined by tower mass and stiffness. Pressure inside the chambers is adjusted according to the velocity and mass of the approaching ship. In this case the main purpose is to avoid direct collision between the ship and the tower and arising excessive forces and accelerations. By using compressed air we also increase the time of impact and possibly mitigate vibrations of the tower induced by ship collision. Inflatable structure for this application can be made of rubber reinforced with metal rods. This kind of material provides adequate durability for high values of pressure and allows large deformation during impact.

Figure 2. Two-dimensional model of inflatable structure for protecting offshore wind turbine.
3. Modelling and control of fluid-filled structures

3.1 Equations governing fluid structure interaction

A precise analysis of the problem described in Sec. 2 requires considering interaction between the walls of the pressurised structure and the fluid contained in its chambers. The main difficulty in such problem is handling the coupling between the deformation of the structure and the pressure exerted by fluid. External load applied to the structure causes its deformation and in effect a change of capacity, temperature and pressure of the fluid. Pressure exerted by the fluid affects, in turn, deformation of the structure and its internal forces.

Let us analyse all equations governing the entire problem of impact applied to the pressurised structure, presented in Fig. 2. The structure is described by a non-linear equilibrium equation (1a) in a static case

$$K(q)q = F(p, q) + F_I$$

(1a)

and a non-linear equation of motion (1b) in a dynamic case

$$Mq + Cq + K(q)q = F(p, q) + F_I$$

(1b)

with initial conditions: $q(0) = q^0$, $\dot{q}(0) = V^0$

where $p = \{p_1(t), p_2(t), ..., p_n(t)\}$ is the vector of gauge pressures in the cavities. The impact can be modelled by r.h.s. force vector $F_I$ or by initial conditions. Anyway, the $F(p,q)$ vector is always present in the problem, since it provides the coupling between the fluid and the structure. Equation of motion has to be considered non-linear since only by assembling the equilibrium equations in actual configuration we can analyse changes of fluid pressure.

We assume that structure is filled with air which is a compressible (pneumatic) fluid with no viscosity. Fluid in each cavity is not discretised into finite elements but described analytically by equation of state for ideal gas:

$$\rho = \rho \cdot R \cdot \theta$$

(2a)

The initial conditions for the fluid are given by: $\rho(0) = \rho_0$, $\theta(0) = \theta_0$ (or two other values chosen from $p, \rho, \theta$).
The absolute pressure $\bar{p}$ is defined as $\bar{p} = p + p_A$, $p_A$ is the ambient pressure, $p$ is the gauge pressure, the absolute temperature is defined as $\bar{\theta} = \theta - \theta_Z$, $\theta$ is the current temperature in Celsius scale and $\theta_Z$ is absolute zero temperature. The gas constant $R$ is related to the universal gas constant $\bar{R}$ and molecular weight MW by the formula: $R = \bar{R}/MW$.

**Remark:** We assume that the pneumatic structure is inflated beforehand and it is in static equilibrium at the initial state. Hence, when applying initial gas pressure we must also introduce initial stresses (and strains) in the structure to fulfill the static equilibrium equation (1a) with force $F_I$ equal zero.

Equation of state (2) is often written in the equivalent form:

$$\rho = \frac{C(p + p_A)}{\theta - \theta_Z} = \frac{(p + p_A)}{R(\theta - \theta_Z)} \quad (2b)$$

where $C$ is constant of the gas given by $C = 1/R = MW/\bar{R}$. Eq. (2) can be also transformed as follows:

$$\bar{p} \cdot V = mR(\theta - \theta_Z) \quad (2c)$$

or $$\bar{p} \cdot V = n\bar{R}(\theta - \theta_Z)$$

where $m$ and $V$ are mass and velocity of the gas and $n$ is number of molecules of the gas. Alternatively, we can obtain:

$$\bar{p} \cdot v = R(\theta - \theta_Z) \quad (2d)$$

where $v$ is the volume of the unit mass (or unit mass) given by: $v = V/m$. Formulation (2b) is often used in ABAQUS, formulations (2c) and (2d) are usually used in classical literature. However, inconsistent notation used by various authors can be misleading.

In the case of fluid flow we also consider conservation of mass given by:

$$m(t) = m_0 + \Delta m(t) \quad (3a)$$

The increase of the fluid mass in the cavity $\Delta m(t)$ is described by integral:

$$\Delta m(t) = \int_0^t q(\bar{t}) d\bar{t} = \int_0^t q_{in}(\bar{t}) - q_{out}(\bar{t}) d\bar{t} \quad (3b)$$
where $q_{in}$ is the mass flow rate into the cavity and $q_{out}$ is the mass flow rate outside the cavity. Direction of the flow is dependent on the sign of the pressure difference. The relation defining the mass flow rate depends on the assumed model of the flow. In the simplest case, it is given by the formulae:

$$\Delta p = C_v q + C_H q|q|$$

where $\Delta p = p_{out} - p$, $C_v$ is the viscous resistance coefficient, $C_H$ is the hydrodynamic resistance coefficient. Mass flow rate can be also dependent on average pressure, average temperature or other fluid parameters.

Under the assumption of an isothermal process (or arbitrarily given change of temperature) a set of equations (1, 2, 3, 4) fully describes the coupling between the fluid and surrounding structure. Navier-Stokes flow equations are not present in the problem formulation and gas dynamics is not involved. The set of equations (1, 2, 3, 4) has to be solved numerically due to nonlinearity of Eq. (1). Upon taking each step, we can calculate the mass flow between the cavities based on the previous fluid parameters. Then, the only additional unknowns (except standard FEM unknowns – $q$) are fluid pressures $p$ in each cavity. For each cavity we also have one additional equation of state which makes the system determinate.

Balance of the gas energy (heat transferred to the system $\Delta Q$, internal energy $\Delta E$ and work done by gas $\Delta W$ is given by the first law of thermodynamics:

$$\Delta E = \Delta Q - \Delta W$$

(5a)

For rapid changes of gas state, adiabatic process which assumes conservation of the gas energy should be considered. When this assumption is applied the temperature change is calculated from the condition that no heat (energy) is added or removed from the cavity except the gas flow between or outside the cavities. Additional equation is equivalent to the first law of thermodynamics:

$$\frac{d(m\overline{E})}{dt} = q_{in} - q_{out} - \dot{W}$$

(5b)

where $\overline{E}$ is specific energy given by:

$$\overline{E} = \overline{E}_1 + \int_{\theta}^\phi c_v(T)dT$$

(5c)
\( \bar{H} \) is specific enthalpy given by

\[
\bar{H} = H_i + \int_0^\theta c_p(T) dT
\]

and \( W \) is work done by gas in the fluid cavity:

\[
dW = \bar{p} dV
\]

The molar heat capacity \( \bar{c}_p \) at constant pressure depends in general on temperature according to the Shomate equation:

\[
\bar{c}_p = \bar{a} + \bar{b} (\theta - \theta_z) + \bar{c} (\theta - \theta_z)^2 + \bar{d} (\theta - \theta_z)^3 + \frac{\bar{e}}{(\theta - \theta_z)^2}
\]

Heat capacity \( c_p \) present in Equation (5d) can be obtained as: \( c_p = \bar{c}_p / MW \). The constant volume molar heat capacity \( \bar{c}_v \) and constant volume heat capacity \( c_v \) can be determined by:

\[
\bar{c}_v = c_v - R; \quad c_v = c_p - R
\]

For the air which is considered in further examples the coefficients in Eq. (6a) are equal: \( MW = 0.0289 \text{kg/mole}; \quad \bar{a} = 28.11 \text{ J/(mole*K)}; \quad \bar{b} = 1.967 \times 10^{-3} \text{ J/(mole*K}^2)\); \( \bar{c} = 4.802 \times 10^{-6} \text{ J/(mol*K}^3\); \( \bar{d} = -1.966 \times 10^{-9} \text{ J/(mol*K}^4)\); \( \bar{e} = 0 \). Anyway, in many practical cases the heat capacity is considered as temperature independent and equal for the air: \( c_v = \frac{5}{2}R, \quad c_p = \frac{7}{2}R \). The equation of energy conservation for adiabatic process (5b) is reduced to:

\[
d[m(\bar{E}_i + c_v(\theta - \theta_i))] = dm(\bar{H}_i + c_p(\theta - \theta_i)) - \bar{p} \cdot dV
\]

Having taken into account that there is no mass exchange in the cavity (\( \Delta m = 0 \)) we obtain:

\[
mc_v \cdot d\theta = -\bar{p} \cdot dV
\]

The above equation can be rewritten in an equivalent form (cf. classical thermodynamics handbooks):

\[
\bar{p} \cdot V^\chi = \text{const.}
\]

where \( \chi = c_p/c_v \) and for the air it is equal: \( \chi = \frac{5}{2}R/\frac{7}{2}R = 1.4 \).
In the case of fluid-structure interaction problem with assumption of adiabatic change of
gas we have to use: equation of motion (1), ideal gas law (2), conservation of mass (3),
flow definition (4) and conservation of the energy (5). The Equations (2–5) apply only
to pneumatic fluids. They are not correct for hydraulic (incompressible) fluids.

3.2 Solutions of typical pneumatic problems

3.2.1 Statics of inflatble structures

Behaviour of fluid elements was investigated precisely in example 2.2.27 from
ABAQUS Verification manual, which provides understanding of basic dependencies
between input data and expected results. In this example, fluid is enclosed within a rigid
chamber with a piston, cf. Fig. 3a. The word “statics” in the example title is used due to
the static equilibrium equation (1a) applied (the inertia of the piston is not taken into
account). Anyway, the volume, pressure and fluid mass have to be considered time
dependent by definition. The example is extended by adding initial force acting on the
piston allowing us to consider various initial pressures in the chamber.

The fluid enclosed in the chamber is subjected to:

0. prestressing: applying initial pressure and initial external force
1. external loading applied to the piston
2. increase of the gas temperature
3. adding a prescribed amount of gas to the cavity
4. removing a prescribed amount of gas from the cavity
5. decrease of the gas temperature.

Each of these conditions is prescribed linearly over subsequent static steps. The purpose
of the analysis is to find the displacement of the piston $u(t)$ and the actual pressure in the
cavity $p(t)$ as a function of initial conditions, as well as external force, increase of temperature and actual mass of the gas in the cavity.

In the situation when stiffness of the structure is not present, a static equation of equilibrium in the actual state (1a) is reduced to a balance of the forces on both sides of the piston:

$$\bar{p}(t)A = p_\Delta A + P_0 + P(t)$$  \hspace{1cm} (8)

Force $P_0$ provides a static equilibrium at the initial state (cf. Fig. 3a) when there is no external force, hence $\bar{p}_0A = p_\Delta A + P_0$ and so $P_0 = p_\Delta A$. The problem is relatively simple since the above equation is not coupled with any other and the pressure inside the piston can be found directly. By introducing ideal gas law (2c) into the mass conservation (3a) we obtain:

$$V(t) = \left(\frac{V_0 \bar{p}_0}{\bar{\theta}_0} + \Delta m(t)R\right) \frac{\bar{\theta}(t)}{\bar{p}(t)}$$  \hspace{1cm} (9a)

By using geometrical relation $u(t) = h_0 - \frac{V(t)}{A}$ we compute piston displacement:

$$u(t) = h_0 - \left(\frac{h_0 \bar{p}_0}{\bar{\theta}_0} + \frac{\Delta m(t) \cdot R}{A}\right) \frac{\bar{\theta}(t)}{\bar{p}(t)}$$  \hspace{1cm} (9b)

In the case when a change of mass in not given explicitly but the fluid exchange is described by Equation (4) with $C_H = 0$ we obtain:

$$u(t) = h_0 - \left(\frac{h_0 \bar{p}_0}{\bar{\theta}_0} - \int_0^t \frac{\bar{p}(t) - p_\Delta d\tilde{t}}{C_\nu} \cdot \frac{R}{A}\right) \frac{\bar{\theta}(t)}{\bar{p}(t)}$$  \hspace{1cm} (9c)

The inverse formula, presented for the sake of completeness, has the form of:

$$\bar{p}(t) = \left(\frac{h_0 \bar{p}_0}{\bar{\theta}_0} + \frac{\Delta m(t) \cdot R}{A}\right) \frac{\bar{\theta}(t)}{h_0 - u(t)}$$  \hspace{1cm} (9d)

By introducing pressure $\bar{p}$ found from (8) into (9b) we finally obtain:

$$u(t) = h_0 - \left(\frac{h_0 \bar{p}_0}{\bar{\theta}_0} + \frac{\Delta m(t) \cdot R}{A}\right) \frac{A\bar{\theta}(t)}{\bar{p}(t) + P_0 + p_\Delta A}$$  \hspace{1cm} (10)
Work done by gas $\Delta W(t)$ is by definition given by $\Delta W(t) = \int_{V_0}^{V(t)} p(t) dV$, which equals:

$$\Delta W(t) = -\int_0^t \bar{p}(t) A d\bar{u} = -\int_0^t \bar{p}(t) A \frac{du(\bar{\theta})}{d\bar{\theta}} d\bar{\theta}$$  \hspace{1cm} (11)$$

By differentiation of $u(t)$ from Eq. (9b) and introducing it into (11) we obtain:

$$\Delta W(t) = \left(\frac{\bar{\theta}(t) - \bar{\theta}_0}{\bar{\theta}_0}\right) h_0 A \bar{p}_0 + \int_0^t \frac{\Delta m(\bar{\theta}(\bar{t})) R \bar{\theta}(\bar{t}) d\bar{t}}{A} + \int_0^t \frac{\Delta m(\bar{\theta}(\bar{t})) R \bar{\theta}(\bar{t}) d\bar{t}}{A} \right) + \int_0^t \frac{h_0 p_0 A \bar{\theta}(\bar{t})}{p_0 A + p(t)} P(\bar{t}) d\bar{t}$$  \hspace{1cm} (12)$$

Above complicated formula is simplified significantly when only one type of excitation is applied simultaneously (as in our problem). For external force, increasing the gas temperature and adding a prescribed amount of fluid into the cavity we obtain respectively:

$$\Delta W_1(t) = h_0 A \bar{p}_0 \ln \frac{p_0 A + P(0)}{p_0 A + p(t)} = V_0 \bar{p}_0 \ln \frac{V(t)}{V_0}$$  \hspace{1cm} (13a)$$

$$\Delta W_2(t) = \left(\frac{\bar{\theta}(t) - \bar{\theta}_0}{\bar{\theta}_0}\right) h_0 A \bar{p}_0$$  \hspace{1cm} (13b)$$

$$\Delta W_3(t) = R \bar{\theta}_0 \Delta m(t)$$  \hspace{1cm} (13c)$$

We notice that the work done by the gas depends here only on final value of the force, its mass and temperature (it is a function of state), which is not the case when all loadings are applied simultaneously. The change of gas internal energy $\Delta E(t)$ can be calculated as a difference of final and initial energy and it equals:

$$\Delta E(t) = m_0 c_v \left(\bar{\theta}(t) - \bar{\theta}_0\right) + \Delta m(t) c_v \bar{\theta}(t)$$  \hspace{1cm} (14)$$

Finally, the energy transferred to the system as heat can be calculated on the basis of the first law of thermodynamics (5a).

The values applied (slightly changed in comparison to Ex. 2.2.27) are, as follows:

- geometry of the model: $h_0 = 1, A = 1$ (area of the piston)
- environment and fluid data: $\theta_e = -460$, $p_A = 14.7$, $\rho_R = 0$, $p_R = 0$, $\theta_R = 200$
- initial state of the system: $\theta_0 = 200$, $p_0 = 12$, $P_0 = 12$ (initial external force)
- loading properties: $P(t) = 88 \ t_1$, $\theta(t_2) = 300 \ t_2$, $\Delta m(t_3) = 15 \ t_3$, $\Delta m(t_4) = -15 \ t_4$, $\theta(t_5) = -300 \ t_5$ (variable $t_n$ indicates time of subsequent time steps).

The values are given in the English metric system. Results presented below (Figs. 3b–3d and Table 1) were obtained by implementing formulae (8–14) into MAPLE software.

Figure 3b. Time history of the loading (force $P(t)$, temperature $\theta(t)$, additional mass $\Delta m(t)$).

Figure 3c. Results of the analysis – gauge pressure in the chamber $p(t)$, piston displacement $u(t)$.

Figure 3d. Results of the analysis – work done by gas $\Delta W(t)$, change of internal gas energy $\Delta E(t)$, energy transferred to the system as a heat $\Delta Q(t)$.
Table 1. Energy components obtained from analytical calculations.

<table>
<thead>
<tr>
<th>Step</th>
<th>$\Delta W(t)$</th>
<th>$\Delta E(t)$</th>
<th>$\Delta Q(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>1</td>
<td>-38.9194</td>
<td>30</td>
<td>-8.9194</td>
</tr>
<tr>
<td>2</td>
<td>-34.8739</td>
<td>40.1136</td>
<td>5.2397</td>
</tr>
<tr>
<td>3</td>
<td>-9.4830</td>
<td>103.5909</td>
<td>94.1078</td>
</tr>
</tbody>
</table>

The results obtained from ABAQUS/Standard are presented below. One can easily check that piston displacement and pressure agree with the values calculated from Equations (8, 10). The energy obtained from ABAQUS (cf. Table 2) is calculated according to formulae different than (13, 14) which are used in classical thermodynamics.

Table 2. Response of the structure at the end of time steps; ALLSE – overall strain energy, ALLWK – external work, ETOT = ALLSE-ALLWK – total energy (ABAQUS output).

<table>
<thead>
<tr>
<th>Step</th>
<th>displ. [u]</th>
<th>pressure [p]</th>
<th>ALLSE</th>
<th>ALLWK</th>
<th>ETOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>12</td>
<td>3.93499</td>
<td>0</td>
<td>3.93499</td>
</tr>
<tr>
<td>1</td>
<td>0.767219</td>
<td>100</td>
<td>31.5763</td>
<td>27.6436</td>
<td>3.9327</td>
</tr>
<tr>
<td>2</td>
<td>0.731949</td>
<td>100</td>
<td>36.3606</td>
<td>24.1166</td>
<td>12.244</td>
</tr>
<tr>
<td>3</td>
<td>0.510581</td>
<td>100</td>
<td>66.3887</td>
<td>24.1166</td>
<td>42.2721</td>
</tr>
</tbody>
</table>

The problem was also solved in the case when all three excitations were applied simultaneously and formula (12) was used for the energy calculations. The example can be also easily extended by applying a ‘fluid link’. In this case, we use definition of mass flow rate instead of prescribed additional mass $\Delta m(t)$. The problem remains relatively simple since the value of pressure in the chamber can obtained directly from Eq. (8).

3.2.2 Flow of the fluid

One of the most typical fluid flow problems is the exchange of fluid between two containers of a given volume, cf. Fig. 4a. Initially, gas enclosed in each of the containers has a different initial pressure. After opening the valve the flow through the link will occur until the pressures in both tanks are equilibrated. Additionally, an arbitrary assumed change of fluid temperature is taken into account. The unknowns addressed in the problem are the change of pressure and mass in both containers and the mass flow rate through the connecting pipe.
The equations governing the problem are the following:

1. ideal gas law applied for initial and actual state of the gas (2c)

\[
p_1^0 V_1 = m_1^0 R \theta_0; \quad p_2^0 = m_2^0 R \theta_0
\]

\[
p_1(t)V_1 = m_1(t)R \theta(t); \quad p_2(t)V_2 = m_2(t)R \theta(t)
\]  

(15)

2. mass conservation in each of the chambers (3a)

\[
m_1(t) = m_1^0 + \Delta m(t)
\]

\[
m_2(t) = m_2^0 - \Delta m(t)
\]

(16)

mass of the fluid transferred by fluid link

\[
\Delta m(t) = \int_0^t \frac{p_2(t) - p_1(t)}{C_v} \, dt
\]

(17)

and the positive \( \Delta m(t) \) indicates the flow from the second chamber to the first one. After substituting mass values found from (15) and (17) into the formula (16) we obtain two integral equations with unknown functions \( p_1(t), p_2(t) \).

\[
\int_0^t \frac{p_2(t) - p_1(t)}{C_v} \, dt = \frac{p_1(t)V_1}{R \theta(t)} - \frac{p_0(t)V_2}{R \theta(t)}
\]

(18a)

\[
\int_0^t \frac{p_2(t) - p_1(t)}{C_v} \, dt = \frac{p_0(t)V_2}{R \theta(t)} - \frac{p_2(t)V_2}{R \theta(t)}
\]

After differentiation with respect to time we obtain two differential equations:
\[
\begin{align*}
\frac{p_2(t) - p_1(t)}{C_V} &= \frac{p_1'(t)\theta(t)}{R\theta(t)} - \frac{p_1(t)\theta'(t)}{R\theta(t)^2} + \frac{p_1V_t\theta'(t)}{R\theta(t)^2} \\
\frac{p_2(t) - p_1(t)}{C_V} &= -\frac{p_2V_t\theta'(t)}{R\theta(t)^2} - \frac{p_2(t)\theta'(t)}{R\theta(t)^2} + \frac{p_2(t)V_t\theta'(t)}{R\theta(t)^2} \\
\end{align*}
\]

(18b)

In the case when the process is isothermal (the temperature is constant) we obtain analytical solution of the problem:

\[
p_1(t) = \frac{p_2^0V_2 + p_1^0V_1}{V_1 + V_2} \left( V_2(-p_1^0 + p_2^0) - \frac{-R\theta(t)_0(V_1 + V_2)}{V_1 + V_2} \right)
\]

(19)

\[
p_2(t) = \frac{p_2^0V_2 + p_1^0V_1}{V_1 + V_2} \left( V_2(-p_1^0 + p_2^0) - \frac{-R\theta(t)_0(V_1 + V_2)}{V_1 + V_2} \right)
\]

In the case when the change of the temperature is given by an arbitrary function, a set of differential equations (18) has to be solved numerically. Mass transported by the fluid link can be calculated directly from (17):

\[
\Delta m(t) = \frac{\partial \theta(t)}{\partial \theta(V_1 + V_2)} \left( p_2^0 + p_1^0 e^{-\frac{\theta(t)_0(V_1 + V_2)}{V_1 + V_2}} - p_1^0 - p_2^0 e^{-\frac{\theta(t)_0(V_1 + V_2)}{V_1 + V_2}} \right)
\]

(20)

Finally, the mass of the gas in each chamber can be obtained from (15) or (16). Changes of pressure and mass of the gas are depicted in Fig. 4b.

The values used for the numerical example are, as follows:

- geometry of the model: \( V_1 = 1, V_2 = 2, C_V = 0.4 \)
- environment and fluid data: \( \theta_z = -460, p_A = 14.7, R = 0.0022272 \)
- initial state of the system: \( \theta_0 = 200, p_0^1 = 80, p_0^2 = 15 \)
- temperature change: \( \theta(t) = 200 + 100t \).

The values are given in the English metric system.
Figure 4b. Results obtained in MAPLE: pressures and mass of the gas in both cavities.

The fluid-link element was also tested using the example 1.3.40 from ABAQUS Verification manual where the chamber with gas is confined by a piston which is connected to a spring, cf. Fig. 5a. The fluid is subjected to the following conditions:

1) constant mass flow rate into the cavity \( q(t) = \text{const.} \)
2) constant pressure at the second end of the link \( p^*(t) = \text{const.} \)
3) increasing pressure in the adjacent container \( p^*(t) \)
4) holding system fix (removing pressure b.c.).

The flow through the link occurs between the fluid cavity and the environment. The unknowns of the problem are: change of volume, change of pressure and flow through the fluid link.

Equilibrium of the piston is given by the algebraic equation:

\[
p(t)A = ku(t)
\]

(21)

\[
p(t)A = k \left[ \frac{V(t) - h_0 A}{A} \right]
\]
Even though the equation is very simple, it couples the unknown pressure and displacement. Moreover, we will use the combination of Equations (2c) and (3a) which gives formulae for $V(t)$, cf. (9a). Finally, our equilibrium is described by:

$$ p(t)A^2 = k \left[ \left( \frac{V_0(p_0 + p_A)}{\theta_0} + \Delta m(t)\bar{R}\bar{\theta}(t) \right) + \frac{1}{p(t) + p_A} - h_0A \right] \tag{22a} $$

In the first step, when the mass flow rate to the main chamber is constant, we substitute: $\Delta m(t) = q \cdot t$ and we obtain the following equation:

$$ A^2 p(t)^2 + (A^2 p_A + h_0k)p(t) + h_0Ak p_A = kV_0(p_0 + p_A) + k\bar{R}\bar{\theta}(t) \cdot qt \tag{22b} $$

which can be easily used for finding pressure $p(t)$ by solving a quadratic equation. New volume of the gas is described by (21). Pressure in the additional container needed to provide constant mass flow rate can be found as:

$$ p^*(\tilde{t}) = p(\tilde{t}) + C_vq \tag{23} $$

For the following, more complicated steps 2 and 3 we have to express mass transferred through the link element by:

$$ \Delta m(t) = \int_0^t \frac{p^*(\tilde{t}) - p(\tilde{t})}{C_v} d\tilde{t} \tag{24} $$

The Equation (22a) assumes the form:

$$ A^2 p(t)^2 + (A^2 p_A + h_0k)p(t) + h_0Ak p_A = kV_0(p_0 + p_A) + k\bar{R}\bar{\theta}(t) \cdot \int_0^t \frac{p^*(\tilde{t}) - p(\tilde{t})}{C_v} d\tilde{t} \tag{25a} $$

which is an integral equation with respect to $p(t)$. After differentiation over time we obtain:

$$ p'(t)\left[2A^2 p(t) + A^2 p_A + h_0Ak\right] = -\frac{k}{C_v}\bar{R}\bar{\theta}(t) \cdot \left[p(t) - p^*(t)\right] \tag{25b} $$

Having solved the above non-linear differential equation, we are able to find the actual gas volume from (9a) and mass flow rate from (23). Step 4 is trivial: after removing a prescribed value of pressure at a free end of the link, the flow is suddenly reduced to zero, pressure in the chamber remains constant and pressure at the end of the link changes suddenly to this value.
One should be careful when superposing the steps 1, 2, 3. The initial values of pressure $p_0$ and volume $V_0$ for each step should be taken from the end of each preceding step.

The values applied for the numerical example are the following:

- geometry of the model: $h_0 = 1, A = 1, C_v = 0.4, k = 1$
- environment and fluid data: $\theta_z = -460, p_A = 14.7, R = 0.00151515$
- initial state of the system: $\theta_0 = 200, p_0 = 0$
- loading properties: $q(t) = 10 t, p^*(t) = 1.84428, p(t) = 3.15572 t$.

The calculations according to (22b) and the solution of the nonlinear ordinary differential equation (25b) were performed in MAPLE software. The results obtained agree with the results from ABAQUS/Standard presented below (Figs. 5b and 5c).

![Figure 5b. Results of the analysis: pressure and volume of the main chamber.](image1)

![Figure 5c. Further results: pressure at free end of the fluid link and mass flow rate.](image2)
3.2.3 Dynamics of inflatable structures

We will consider a cylinder confined by a piston, as in Sec. 3.2.1. This time, the inertia of the piston will be taken into account and the equation of motion (1b) will be used to describe the equilibrium of the system.

![Figure 6a. Cylinder with the piston subjected to dynamic loading.](image)

The fluid in the piston will be subjected to simultaneous conditions as in static case: i) external loading applied to the piston, ii) increase of the gas temperature, iii) the displacement of the piston. Actual pressure will be found in terms of arbitrarily given functions $P(t)$, $\theta(t)$ and $\Delta m(t)$. Equation (1b) with the structure stiffness and damping neglected is reduced to:

$$ M\ddot{u}(t) + \bar{p}(t)A = p_A A + P_0 + P(t) \quad (26a) $$

The force $P_0$ provides a static equilibrium at the initial state when there is no external force and no acceleration of the piston: $P_0 = p_0 A$. The above equation is much more complicated than Equation (8) for a static case since it combines gas pressure and piston displacement. By introducing function $\bar{p}(t)$ according to (9d) we obtain:

$$ M \dddot{u}(t) + \left( A h_0 \frac{\bar{p}_0}{\bar{\theta}_0} + \Delta m(t) \cdot R \right) \frac{\bar{\theta}(t)}{h_0 - u(t)} = p_A A + P_0 + P(t) \quad (26b) $$

$$ \dot{u}(0) = V_0 $$

The above nonlinear ordinary differential equation has to be solved numerically in order to find piston displacement $u(t)$.

After integrating (26a) over time we obtain conservation of the impulse:
\[ \int_0^t M \cdot \frac{d^2 u(t)}{dt^2} \, dt + \int_0^t A \ddot{p}(t) \, dt = \int_0^t \ddot{A}P_0 + P(t) \, dt \]  

(27a)

By writing this equation for the time \( t = t' \) when \( \dot{u}(t) = 0 \) we obtain:

\[ -M \cdot V_0 + \int_0^{t'} A \ddot{p}(t) \, dt = \int_0^{t'} \ddot{A}P_0 + \int_0^{t'} P(t) \, dt \]  

(27b)

Equation (27b) can be used to calculate the impulse transmitted to the system. Integrating (26a) over displacement gives conservation of the energy which reads:

\[ \int_0^u M \cdot \frac{d^2 u(t)}{dt^2} \, du + \int_0^u \ddot{P}(t) Adu = \int_0^u \ddot{P}_0 Adu + \int_0^u P(t) du \]  

(28a)

or \( \Delta E_k(t) - \Delta W(t) = W_{EXT}(t) \)  

(28b)

where \( \Delta E_k(t) \) is change of piston kinetic energy and \( W_{EXT}(t) \) is work done by external forces. The work done by gas \( \Delta W(t) = -\int_0^u \ddot{P}(t) Adu \) can be calculated by using Eq. (28):

\[ \Delta W(t) = \frac{1}{2} MV(u(t))^2 - \frac{1}{2} MV_0^2 - \int_0^u \dddot{P}_0 A\dot{u}(t) \, dt - \int_0^u P(t)u'(t) \, dt \]  

(29a)

or by using the formula defining pressure (9b):

\[ \Delta W(t) = \int_0^t \left( \frac{h_0 \ddot{P}_0}{\ddot{\theta}} + \frac{\Delta m(t) \cdot R}{\theta} \right) \dddot{\theta}(t) \, dt - \int_0^t \frac{h_0}{\ddot{u}(t)} \dddot{u}(t) \, dt \]  

(29b)

Unfortunately, calculation of each energy component has to be preceded by solving differential equation, since values \( u(t), V(u(t)), \theta(t) \) and \( \Delta m(t) \) are required. Hence, the explicit formula for work done by gas (by analogy to (12)) can not be given.

Internal energy of the gas \( \Delta E(t) \) can be calculated in the same manner as in the static case, according to classical formula (14). The energy transferred to the system by heat \( \Delta Q(t) \) can be calculated according to the first law of thermodynamics (5a).

The values applied (the same as in prepared experiment) are the following:

- geometry of the model: \( h_0 = 0.15 \, m, A = 0.002827 \, m^2, M = 1 \, kg \)
– environment and fluid data: $\theta_e = -273.15 \, ^\circ C, p_A = 10e5 \, Pa, R = 287.69 \, J/kg/K$
– initial state of the system: $\theta_0 = 20 \, ^\circ C, p_0 = 30e5 \, Pa, V_0 = 4.42 \, m/s$.

Numeric solution of the nonlinear differential equation (26b) was done in MAPLE software by using Fehlberg fourth-fifth order Runge-Kutta method (built-in `dsolve` command, `rkf45` option). The results are presented for $P(t) = 0 \, d$ for the three following cases:

1. isothermal process without flow of the gas $\theta(t) = \theta_0, \Delta m(t) = 0$
2. influence of arbitrary assumed temperature change $\theta(t) = 20 + 500 \, t$
3. influence of removing assumed amount of gas during analysis $\Delta m(t) = -0.006 \cdot t$.

![Figure 6b. Displacement of the piston: isothermal process without outflow (black line), influence of temperature increase (red line), influence of the gas removal (blue line).](image)

The main conclusions from the above results are the following. Air chamber has larger stiffness during compression than during tension (compare amplitudes in Fig. 6b). The increase of temperature causes that the average distance between the bottom of the chamber and the piston in subsequent cycles is rising. Removing fluid from the cavity results in an increase of frequency of piston vibrations and causes the piston movement inside the chamber. The results presented in Fig. 6c and 6d apply to the situations 2 and 3 occurring simultaneously (the temperature of the gas is rising and the fluid is removed from the cavity). These results agree very precisely with the results obtained from ABAQUS/Standard analysis.

![Figure 6c. Results of the analysis – gauge pressure in the chamber $p(t)$, piston displacement $u(t)$.](image)
A short interpretation of these results is, as follows: due to the fact of removing fluid, the piston is approaching the bottom of the cylinder. The pressure difference between the chamber and environment is higher so the piston is moving faster and its maximal kinetic energy is increasing. Work done by gas is negative since the piston is approaching the bottom of the chamber. Internal gas energy is decreasing because the influence of gas removal is stronger than the effect of the temperature increase. Initially, the energy is transferred to the system, but finally it is dissipated from the system.

**Isothermal and adiabatic process**

General equation (26b) can be specified to the case of isothermal or adiabatic process. In isothermal process, the temperature of the gas remains constant. It usually happens when the process is relatively slow and the temperature of the gas is constant and equal to environment temperature. In the case when the mass of the gas is constant, there is no change of the internal gas energy, cf. (14) and all work done by gas is changed into heat transferred from the system: 

\[ \Delta W = -\Delta Q. \]

Adiabatic process occurs when changes of the gas state are very fast and no energy exchange between the gas and the environment occurs. In the case when the mass remains constant, the condition (5b) means that work done by gas is changed into its internal energy. This condition can be also written as 

\[ \Delta Q(t) = 0 \text{ or } \frac{\partial}{\partial t}(\bar{p} \cdot V^x) = \text{const.} \],

cf. (7c). Temperature in the adiabatic process is calculated from the ideal gas law (2d) and adiabatic equation (7c) which yields:
\[ \bar{\theta}(t) = \bar{\theta}_0 \left( \frac{V_0}{V(t)} \right)^{1/x} \quad \text{or} \quad \bar{\theta}(t) = \bar{\theta}_0 \left( \frac{\bar{p}_0}{\bar{p}(t)} \right)^{1-x} \]  

(30a,b)

We can find the relation between the pressure and chamber volume by introducing temperature from (30b) into (9a) obtaining:

\[ V(t) = \left( V_0 \bar{p}_0^{1/x} + \Delta m(t)R\bar{\theta}_0 \bar{p}_0^{1/(x-1)} \right) \bar{p}(t)^{-1/x} \]  

(31a)

The above procedure may seem weird since Equations (30a, b) were derived basing on (7c), which assumes that the mass of the gas remains constant and Equation (9a) takes into account the change of the gas mass. The explanation is that (30b) concerns ‘intensive’ thermodynamics values which are not dependent on fluid volume. In contrast, (30a) concerns ‘extensive’ thermodynamics values and cannot be used for deriving volume – pressure relation. By using geometrical relation \( V(t) = (h_0 - u(t))A \) we obtain piston displacement:

\[ u(t) = h_0 - \left( h_0 A\bar{p}_0^{1/x} + \frac{\Delta m(t)R\bar{\theta}_0 \bar{p}_0^{1/(x-1)}}{A} \right) \bar{p}(t)^{-1/x} \]  

(31b)

In the case when flow of the fluid is described by Equation (4) with hydraulic resistance coefficient equal to zero we obtain:

\[ u(t) = h_0 - \left( h_0 A\bar{p}_0^{1/x} - \frac{\int_0^t \bar{p}(t) - \bar{p}_a dt}{C_y} \right) \bar{p}(t)^{-1/x} \]  

(31c)

The formula inverse to (31a) yields:

\[ \bar{p}(t) = \left[ \frac{1}{h_0 - u(t)} \left( h_0 A\bar{p}_0^{1/x} + \frac{\Delta m(t)R\bar{\theta}_0 \bar{p}_0^{1/(x-1)}}{A} \right) \right]^x \]  

(31d)

In formula (31) we assume that the gas entering the cavity has the same temperature as gas in the chamber. This assumption is necessary to avoid the energy exchange between these two gases (to provide the conditions of adiabatic process).

After introducing pressure from (31d) into (26) we obtain final equation defining the piston displacement.
\[ M \dddot{u}(t) + A \left[ \frac{1}{h_0 - u(t)} \left( h_0 \bar{p}_0^{1/\chi} + \frac{\Delta m(t) \bar{\rho}_0 \bar{p}_0^{1/\chi - 1}}{A} \right) \right]^\chi = p_4 A + p_0 + P(t) \]  

(32a)

In a simple case when there is no flow of the gas the above equation is simplified to:

\[ M \dddot{u}(t) + A \bar{p}_0 \left( \frac{h_0}{h_0 - u(t)} \right)^\chi = p_4 A + p_0 + P(t) \]  

(32b)

Equation of motion can be integrated over the displacement according to (28). The work done by gas equals to \( \Delta W(t) = \Delta E_{Ax}(t) - W_{Ex}(t) \) or \( \Delta W(t) = \int_0^u \bar{p}(t) A d\bar{u} \) which reads:

\[ \Delta W(t) = \int_0^t \left[ \frac{1}{h_0 - u(t)} \left( h_0 \bar{p}_0^{1/\chi} + \frac{\Delta m(t) \bar{\rho}_0 \bar{p}_0^{1/\chi - 1}}{A} \right) \right]^\chi u'(\bar{t}) d\bar{t} \]  

(33a)

The above integral can be calculated analytically for the case when no additional gas is added to the piston, and it equals:

\[ \Delta W(t) = \bar{p}_0 A h_0^\chi \frac{1}{\chi - 1} \left[ (h_0 - u(t))^{-\chi + 1} + h_0^{-\chi + 1} \right] \]  

(33b)

When applying the relation between the gas constant and constant volume heat capacity \( R = c_v (\chi - 1) \) and formula (30a) we notice that we can as well write:

\[ \Delta W(t) = \bar{p}_0 A h_0^\chi \frac{c_v}{R \bar{\rho}_0} \left[ \frac{h_0 - u(t)}{h_0} \right]^{-\chi + 1} + 1 = -m c_v (\bar{\theta}(t) - \bar{\theta}_0) = -\Delta E(t) \]  

(33c)

which proves that all work done by gas is changed into internal energy of the gas. By introducing work calculated in (33b) into (28a) applied for the case with no external force and written for a time of stopping the piston we obtain the following:

\[ -1/2 M \cdot V_0^2 + \bar{p}_0 A h_0^\chi \frac{1}{\chi - 1} \left[ (h_0 - u_{\text{max}}) \right]^{-\chi + 1} - h_0^{-\chi + 1} = \bar{p}_0 A u_{\text{max}} \]  

(34)

This equation can be effectively used for calculating:

- maximal piston displacement as a function of impact energy and initial pressure
– initial pressure needed to achieve a desired piston displacement for assumed impact energy
– initial pressure as a function of impact energy when allowable piston displacement is given.

Equations for the isothermal process relating the gas pressure and its volume, as well as equation of motion can be easily obtained from (9, 26) by introducing $\theta(t) = \theta_0$ or, alternatively, from (31, 32) by introducing $\chi = 1$. In (33b, c; 34) we cannot apply $\chi = 1$ directly, but we have to calculate $\lim_{\chi \to 1}$. We obtain the work done by gas:

$$\Delta W(t) = -\overline{p}_0 \overline{h}_0 \overline{A} \ln \left( \frac{h_0}{h_0 - u(t)} \right)$$

(35)

and the conservation of the energy for isothermal process:

$$-1/2M \cdot V_0^2 + \overline{p}_0 \overline{h}_0 \overline{A} \ln \left( \frac{h_0}{h_0 - u_{\text{max}}} \right) = -\overline{p}_0 \overline{A} u_{\text{max}}$$

(36)

Formula (36) can be used similarly as (34) for the purpose of calculating maximal piston displacement for a given impact energy.

The values used in the numerical example are the following (the problem like in Fig. 6a, but with the isothermal and adiabatic change of gas assumed):

– geometry of the model: $h_0 = 0.15 \text{ m}, A = 0.002827 \text{ m}^2, M = 1 \text{ kg}$
– environment and fluid data: $\theta_z = -273.15 \text{ C}, p_A = 10 e 5 \text{ Pa}, R = 287.69 \text{ J/kg/K}$
– initial state of the system: $\theta_0 = 20 \text{ C}, p_0 = 30 e 5 \text{ Pa}, V_0 = 4.42 \text{ m/s}$
– gas removed from the cavity $\Delta m(t) = -0.004 \cdot t$.

Figure 6e. Results of the analysis – gauge pressure in the chamber $p(t)$, piston displacement $u(t)$ (blue line – isothermal process, red line – adiabatic process).
Results of the analysis are presented in Fig. 6e, 6f. The increase of temperature in adiabatic process is significant, it decreases below zero Celsius and increases up to 80 degrees. The increase in temperature is strongly dependent on the mass of fluid removed from the cavity. However, the difference in pressure and displacements amplitude is only several percent. Moreover, in adiabatic process we observe an increase of piston vibrations frequency. The above conclusions are relevant only for the presented example. Differences in solutions of the isothermal and adiabatic process may vary significantly depending on the problem considered.

**Flow through the orifice**

Additional mass introduced into the system may be not given arbitrarily, but mass flow rate may be dependent on pressure difference according to (4). When hydrodynamic coefficient is equal to zero, additional mass is given by:

\[
\Delta m(t) = \int_0^t \frac{q(t)}{C_r(t)} \, dt = \int_0^t \frac{p(t) - \bar{p}(t)}{C_r(t)} \, dt
\]  

and set of equations defining the system is composed of equilibrium equation and relation between piston displacement and pressure:

\[
M \ddot{u}(t) + \bar{p}(t)A = \bar{p}_0A + P(t)
\]  

\[
u(t) = h_0 - \left( h_0 \bar{p}_0 \frac{\bar{p}(t) - p(t)}{A} \right) \frac{R}{A} \left( \frac{\bar{\theta}(t)}{\bar{p}(t)} \right)
\]

with initial conditions: \( u(t) = 0, \ \dot{u}(t) = V_0, \ \bar{p}(t) = \bar{p}_0 \)

The first one is a differential equation with respect to \( u(t) \) and the second is an integral equation with respect to \( p(t) \). Moreover, these two equations are coupled since functions.
\( p(t) \) and \( u(t) \) are present in both of them. One equation describing the whole system can be obtained by differentiating the second equation over the time twice and introducing found acceleration into the equilibrium equation. This way we obtain one differential-integral equation. Unfortunately, the mathematical software available was not able to numerically solve the set of equations (38) nor one differential-integral equation.

However, we can transform (38b) into differential equation by calculating first derivative after leaving only the integral on l.h.s.

\[
\frac{A u(t) \overline{p}(t)}{R \overline{\vartheta}(t)} - \frac{A h_0 \overline{p}(t)}{R \overline{\vartheta}(t)} + \frac{h_0 p_0}{\theta_0} = \int_0^t \frac{\overline{p}(\tilde{t}) - p_A}{C_v} d\tilde{t}
\]

(39a)

\[
\frac{\overline{p}(\tilde{t}) - p_A}{C_v} = A \frac{\overline{p}(t) u'(t)}{R \overline{\vartheta}(t)} - A \frac{(h_0 - u(t)) \overline{p}(t) \overline{\vartheta}(t) - \overline{p}(t) \overline{\vartheta}(t)}{R \overline{\vartheta}(t)}
\]

(39b)

The set of equations (39a, b) can be solved numerically. The values of pressure and its first derivative obtained from (39a) can be as well introduced to (39b) to obtain one equation of the third order (here given for the case of isothermal process without force excitation):

\[
- \frac{M u'(t) u''(t)}{R \overline{\vartheta}_0} + \frac{M (h_0 - u(t)) u'''(t)}{R \overline{\vartheta}_0} + \frac{A \overline{p}_0 u'(t)}{R \overline{\vartheta}_0} = \frac{M u'''(t)}{A C_v} + \frac{p_0 - p_A}{C_v}
\]

(40)

with the initial conditions as in (38) and one additional initial condition on \( u''(t) \) found from the equilibrium equation (38a) and equal to: \( \ddot{u}(0) = \frac{P(0)}{M} \). The formula defining work done by gas (29) remain valid.

The numerical example was done for the same geometry of the model, environment and fluid data and initial state of the system, but the process was assumed as isothermal. Excitation force was increasing with time according to: \( P(t) = 20000 t^3 \) and values of viscous resistance coefficient were chosen as: \( C_v^1 = 0,4e8 Pa \cdot s / kg \) (the valve partially open – red line), \( C_v^2 = 0,24e8 Pa \cdot s / kg \) (the valve almost fully open – blue line).

![Graph](image)

**Figure 6g. Pressure inside the chamber, piston displacement.**
The values of the flow resistance coefficient were chosen in a way that about 60% of the fluid is removed from the cavity in the first case and almost whole fluid is removed in the second case. For lower values of the resistance flow coefficient the solution becomes numerically unstable. We can observe once again that the piston vibration frequency is increasing when the fluid is removed, cf. Fig. 6g. However, the amplitude of pressure in both cases is similar. The mass flow rate changes as pressure does, but is scaled by flow resistance coefficient, cf. Fig. 6h. In the second case, the amplitude of the piston vibrations is decreasing significantly. The piston almost reaches the bottom of the chamber at the end of the analysis.

**Remark:** One should take into account that real change of gas is neither isothermal nor adiabatic, but something in-between. Assumptions of isothermal and adiabatic process can be treated as extreme solutions of the considered problem.

### 3.3 Modelling of inflatable structures in ABAQUS

In ABAQUS software there are two main possibilities of modelling above described fluid-structure interaction: using fluid-filled cavities and using surface-based cavities. Two additional options are: equation of state and fully coupled fluid-structure interaction.

#### 3.3.1 Fluid-filled cavities

The capability of fluid-filled cavities is available in ABAQUS/Standard and Explicit. It provides the coupling between structure and the enclosed fluid. ‘Fluid-filled cavities’-based modelling does not allow adiabatic change of the gas which means that Equation (5b) is not taken into account. Fluid pressure and temperature may vary in time (during analysis) but they are uniform at every location of the cavity. It is also assumed that the cavity is completely filled with pneumatic (compressible) or hydraulic (incompressible) fluid.
To model fluid-structure by fluid-filled cavities we use:

- standard finite elements to model the structure
- hydrostatic fluid elements for coupling between the deformation of the fluid-filled structure and the pressure exerted by fluid
- definition of the hydrostatic fluid properties
- fluid link elements to transfer fluid between the cavities and the outer surface (cf. 3.3.3).

Hydrostatic fluid elements have to cover only a boundary of the fluid-filled cavity. This means that for 3D structure we use 2D fluid elements and for 2D structure we use 1D fluid elements. Fluid elements share the nodes with standard elements. It is also possible to define fluid element between the nodes where no standard element exists. All fluid elements are first-order which means that the interpolation of the element displacement is linearly dependent on node displacement. This linear interpolation is used for calculating the volume of the fluid cavity. The period normal to the fluid element must point into the fluid. The fluid elements available in ABAQUS are: F2D2 (used in further analysis), F3D4 and FAX2.

All hydrostatic fluid elements associated with the cavity share common reference node which has a single degree of freedom representing pressure in the cavity. The location of the cavity reference node is arbitrary except for an axisymmetrical case. If there are several fluid cavities within the structure each of them has a separate reference node. Actual pressure in the cavity (PCAV) and actual cavity volume (CVOL) can be obtained as output in the cavity reference node.

Properties of the fluid are defined by the following data:

- absolute zero temperature defined by *PHYSICAL CONSTANTS
- type of the fluid (hydraulic or pneumatic) defined by *FLUID PROPERTY
- thickness of the fluid cavity for 2D case defined by *FLUID PROPERTY
- reference density, reference pressure and reference temperature of the fluid defined by *FLUID DENSITY.

Basing on these values, ABAQUS will calculate gas constant \( C \) (or \( R \)) using (2b). Separate defining of the universal gas constant \( R \) is not required.

**Initial and boundary conditions**

The following initial conditions for the fluid must be specified at the beginning of the analysis according to formula (2a):
– initial pressure of the gas by *INITIAL CONDITIONS, TYPE=PRESSURE
– initial temperature by *INITIAL CONDITIONS, TYPE=TEMPERATURE.

**Remark:** pressure inside the cavity is always considered as gauge pressure \( p \). Hence, both initial conditions and output are expressed in terms of gauge pressure. On contrary pressure considered in ideal gas law (2c), (2d) is always the absolute (total) pressure.

Using given initial conditions, the value of constant \( C \) and the value of initial cavity volume calculated beforehand, ABAQUS is able to calculate initial mass of the gas in the cavity as:

\[
m_0 = \rho_R (\theta_a - \theta) V_v (p_a + p_d) = \frac{V_v (p_a + p_d)}{R (\theta_a - \theta)} = V_v p_0
\]  

(41)

Temperature of the fluid in the cavity can also be specified as a predefined field by applying *TEMPERATURE command. Then equation of state for ideal gas can be verified by using the equality:

\[
\frac{V_v (p_a + p_d)}{\theta_a - \theta} = \frac{V(t) V(t) + p_d)}{\theta(t) - \theta}
\]  

(42)

Equation (42) is valid when the mass of the gas is kept constant, regardless of the type of process (isothermal, adiabatic). In the case when the change of temperature is not given as input, isothermal process is assumed by default and the temperature remains constant during the whole analysis.

The pressure degree of freedom (number 8) at the cavity reference node is a primary variable of the fluid-structure interaction problem. Thus, the boundary condition can be prescribed to the pressure degree of freedom. In such case the fluid mass is automatically adjusted to fill the cavity (fluid is assumed to enter and leave the cavity as needed to maintain the prescribed pressure). To avoid the jump of the pressure at the beginning of the analysis it is possible to introduce initial condition imposed on pressure.

As it was mentioned in Sec. 3.1, initial conditions of the system must provide its equilibrium. However, in more sophisticated cases it is not obvious what is the stress/strain state caused by initial pressure. Assuming no stress and strain with nonzero initial pressure leads to correct results in ABAQUS/Standard and unexpected, unreasonable results in ABAQUS/Explicit. This approach is not recommended.

Alternatively, we can divide the analysis into two steps. In the first step we apply pressure boundary condition in which we increase the pressure gradually from zero to
assumed value. The length of this step should be adjusted to inflator properties. The
time of the step can be also close to zero, if needed. In the next step we can remove the
pressure boundary condition and apply external forces. This second approach fulfills
initial equilibrium and gives similar reasonable results in both used solvers.

The above described features of the fluid-filled cavities applied for the inflatale
structure problem described by Equations (1, 2) and depicted in Fig. 2 allow us to:

− model structure filled with prescribed initial pressure
− model structure with pressure and temperature arbitrary changing during impact
− define inflating of the cavity
− apply FORTRAN subroutine DISP for controlling pressure boundary condition.

Flow of the fluid

The easiest possibility of modelling flow of the fluid is just removing and adding the
fluid into the cavity using *FLUID FLUX. Using this command we define mass flow
rate \( q \) into the cavity. Defined flux can vary during the step when *AMPLITUDE
command is used.

A non-direct flow of the fluid is performed when pressure boundary condition is
defined. The amount of fluid in the cavity is automatically adjusted to fulfill applied
pressure boundary condition, a given temperature and calculated volume of the cavity.
The mass of fluid can be derived from Equation (2) and is equal to:

\[
\Delta m = \frac{V(t) \cdot (p(t) + p_A)}{R(\theta(t) - \theta_2)} - \frac{V_0(p_0 + p_A)}{R(\theta_0 - \theta_2)}
\]  

(43a)

By using the density definition the above formula can be transformed to:

\[
\Delta m = \rho \cdot \Delta V + \rho_0 V_0 \left(\frac{\rho}{\rho_0} - 1\right)
\]

(43b)

In a trivial case when \( p = p_0, \theta = \theta_0 \) which implies \( \rho = \rho_0 \) formula (43b) is simplified to:

\[
\Delta m = (V - V_0) \rho_0
\]

(43c)

The third and the most sophisticated (when using fluid-filled cavities) method of
modelling fluid flow is using *FLUID LINK elements. Fluid link elements are defined
between the cavity reference nodes. The mass flow rate through the link can be defined
as a function of pressure difference according to (4). Flow rate through the link can be
also dependent on average pressure, average temperature, and external field variables which can be changed during the analysis by *AMPLITUDE command. Other possibility then applying formula (4) is a tabular definition of \(q\) versus pressure and temperature. In ABAQUS/Standard there is no parameter describing the area of the valve so the flow intensity can be adjusted only by changing coefficients \(C_v\) and \(C_H\). Fluid transfer between a cavity and the environment can be modelled if one side of the link is not connected and the pressure at that end is prescribed with a boundary condition. Output of the fluid link element contains mass flow rate (MFL) and total mass flow (MFLT).

**Remark:** Using both fluid link element and pressure boundary condition in application to one fluid-filled cavity is not recommended. The flow of the gas will be, in this case, dependent only on pressure difference. The change of the amount of fluid due to applied pressure boundary condition is not taken into account when calculating flow through the fluid link.

Described features of the fluid-link element applied for the case of ship hitting inflatable structure allow us to:

- model fluid-filled structure with possibility of a pressure-dependent fluid flow
- open and close the valve and change properties of the fluid flow during analysis
- apply FORTRAN subroutine UFIELD for changing properties of the flow according to the actual results of the analysis.

**Example**

We will analyse how to model various pneumatic structures by changing the properties of the fluid link element. The following conditions of the fluid-filled structure may occur:

0. structure without internal gas
1. structure with internal gas without possibility of any flow
2. constant pressure of the gas inside the structure during analysis
3. flow of the gas outside the structure or/and between the chambers.

All these cases can be modelled by introducing fluid link elements between the cavity and the environment and by adjusting value of the flow coefficients and value of external pressure. We may deal with the following situations:

a. \(C_v = \infty\) then \(q = 0\) and there is no flow of the gas (case 1)
b. \(C_v = 0\) then \(p = p_{\text{out}}\) and the pressure is constant inside the structure
   - assuming \(p_{\text{out}} \neq 0\) is artificial, however effectively implements case 2
   - assuming \(p_{\text{out}} = 0\) is equivalent to modeling structure without gas (case 0)
c. \(C_v = (0, \infty)\) then the flow inversely proportional to the value of \(C_v\) occurs (case 3).
These three possibilities were implemented for the problem of ship colliding into wind turbine presented in Fig. 2. In the first plot $C_V = \infty$ so the chamber is closed (Fig. 7a). We can observe the increase of pressure when the chamber is compressed and the reverse phenomenon when it is expanded. The product of absolute pressure and volume remains constant during the analysis but this condition is not fulfilled exactly.

![Figure 7a. Change of pressure, volume and ideal gas law in case 1 (no gas flow).](image)

The second plot presents the case when pressure remains constant during the impact (Fig. 7b). Due to this fact, the changes in chamber volume are a little larger. The amount of gas in the cavity is adjusted by ABAQUS to fulfill the constant pressure condition. The mass of fluid introduced into the system can be found from (43).

![Figure 7b. Change of pressure, volume and flow of the gas in case 2 (constant pressure).](image)

The last plot concerns the case when the valve is open (Fig. 7c). Pressure inside the chamber and its volume are significantly decreasing at the end of analysis. The flow of the gas $q(t)$ is proportional to the pressure difference so its plot is similar to pressure plot.
3.3.2 Surface-based cavities

The capability of “surface-based cavities” is available only in ABAQUS/Explicit. Similarly as “fluid-filled cavities” it provides coupling between the structure and the fluid enclosed inside. The fluid is described by Equations (2, 3, 4, 5) so modelling adiabatic change of gas is possible. Both pneumatic and hydraulic fluids can be used during the analysis.

To model fluid-structure interaction using surface-based cavities we use:

− standard finite elements to model the structure
− surface definition on the cavity boundary for coupling between the deformation of the structure and pressure exerted by the fluid
− definition of the fluid behaviour
− fluid exchange definition to model flow of the fluid
− fluid inflator to model deployment of the cavities.

In present case the fluid cavity is defined by confining surface not by fluid elements, like in Sec. 3.3.1. This difference is rather formal since in both cases the fluid cavity is not discretised but described as a whole. Surface can be node-based or element-based type. To model surface in the region where there are no standard elements are present surface elements can be applied. Each cavity has the reference node in which fluid pressure, volume and temperature is stored.

Properties of the fluid are defined in different way than in case of “fluid-filled cavities” and are described by the following data:

− absolute zero temp. and universal gas constant by *PHYSICAL PROPERTIES
- type of gas change (isothermal or adiabatic) by *FLUID CAVITY
- constant pressure heat capacity by *CAPACITY
- molecular weight by *MOLECULAR WEIGHT
- grouping features of gas and cavity by *FLUID BEHAVIOUR, *FLUID CAVITY.

The initial and boundary conditions can be applied in the same manner as in case of fluid-filled cavities. The output for surface-based cavities contains fluid pressure, volume, temperature, mass and cavity surface area (PCAV, CVOL, CTEMP, CMASS, CAREA).

Flow of the gas between the cavities and outside the structure is defined by *FLUID EXCHANGE and *FLUID EXCHANGE PROPERTIES. Fluid link elements between the reference nodes of the cavities are not used. Gas flow can be based on the following mechanisms: mass-based flux, volume-based flux, adiabatic flow through the exhaust orifice, fabric leakage. The mass or volumetric flow rate versus pressure difference can be specified as a table. Effective area of the flow may be given. The flow can be dependent on “field variables”. It can be also varied during the time step by using the *AMPLITUDE command.

The process of the cavity inflation can be modelled by using tank test data, dual pressure method or specifying inflator pressure and mass flow rate directly. For all this cases the detailed data from the experiment are required. Due to the lack of this data the method of applying pressure boundary condition remains the most appropriate for a wide variety of problems.

Single physical chamber may be divided by means of fictitious walls to model the pressure difference between regions of the chamber. The variety of the flow mechanisms provides appropriate choice of the artificial leakage through the inner-chamber wall. Obtaining the average results for multi-chamber model is also possible.

**Comparison to fluid-filled cavities**

The results of the analysis found by using “fluid-filled cavities” and “surface-based cavities” were exactly the same when using ABAQUS/Explicit. When using ABAQUS/Standard for “fluid-filled” cavities small inconsistencies may appear due to differences of implicit and explicit scheme of the solution. Especially in ABAQUS/Standard analysis, the small decrease of the pressure and volume after removing pressure boundary conditions occurs.
In general using “surface-based cavities” in ABAQUS/Explicit was found to be more convenient according to following features:

- possibility of straightforward modelling of the adiabatic process
- various options for fluid flow including Saint-Venant formula for adiabatic flow of the gas through the orifice
- possibilities of precise modelling of the inflation process
- option of modelling ideal gas mixture and properties of the multiple cavities
- better output for the fluid cavities including mass of the fluid
- no problems with the convergence of the analysis
- shorter time of the analysis.

In all cases when using the subroutines was not necessary the surface based cavities implemented in ABAQUS/Explicit were used. However, the main disadvantage of the “surface-based cavities” is that they are available only in ABAQUS/Explicit where user subroutines (URDFIL, UFIELD, DISP, UTEMP, UFLUID) for controlling properties of the fluid during the analysis are not available.

When active control of the fluid parameters or fluid flow was needed fluid filled cavities (ABAQUS/Std) were used and lacking features were implemented by FORTRAN subroutines. The other possibility is overcoming the disadvantages of ABAQUS/Standard is preliminary run the analysis in ABAQUS/Explicit and then import of necessary results (for instance temperature) into ABAQUS/Standard as a predefined field.

### 3.3.3 Equation of state

In the case when spatially varied pressure and temperature inside a particular cavity is required it is possible to use “Equation of state (EOS)” option. Standard finite elements have to be applied for modelling of fluid domain. In the case of our problem (cf. Fig. 2) the most appropriate are shell elements or three dimensional plane strain elements. Material properties are defined by equation of state (2a) by using the command *EOS, TYPE=IDEAL GAS. This capability is available only in ABAQUS/Explicit. Parameters of the gas-like gas constant, ambient pressure, molecular weight and constant pressure heat capacity has to be defined.

Initial state of the gas is defined by:

- initial density: *DENSITY
- initial pressure stress: *INITIAL CONDITIONS, TYPE=STRESS
- initial temperature: *INITIAL CONDITIONS, TYPE=TEMPERATURE.
The “equation of state” approach to coupled fluid-structure problems was not tested. As the method involving discretisation of the fluid the computational cost is expected to be higher. It is planned to use this option to check the variation of the pressure and temperature across the cavity and then adjust the values of the flow coefficients between the fictitious chambers modelled using “surface-based elements”.

### 3.3.4 Fully coupled fluid-structure analysis

The most sophisticated method of modelling fluid-structure interaction is solving equilibrium equations for the structure and Navier-Stokes equations for the fluid. This method uses advantages of Lagrange description of structure deformation and Euler description of fluid deformation and is called Arbitrary Lagrangian Eulerian (ALE) formulation.

To implement this solution both the structural finite element code and computational fluid dynamics (CFD) code has to be used. For this purpose ABAQUS can be linked to FLUENT software. At the moment this fully coupled method of solution seems to be too complicated for the considered problem. In fact the exact values of the displacements and velocities of each fluid particle are not of interest. Fluid part of the analysis is important to provide appropriate fluid-structure coupling conditions and transmit forces into the structure at the boundary of the cavity.

**ABAQUS examples**

The details of modelling fluid-filled cavities are described in ABAQUS User’s Manual p. 7.12, hydrostatic fluid properties in p. 12.4 and hydrostatic fluids elements in p. 18.8. The most suitable ABAQUS examples concerning hydrostatic fluid elements are: Example problem manual: Modelling an air spring (1.1.8), Tennis ball and rocket (2.1.5); Verification Manual: Fluid link element (1.3.40), Hydrostatic fluid elements (2.2.27); Benchmark Manual: examples: 1.1.7, 2.5.1, 1.13.11, 1.13.15.

### 3.4 Additional features implemented by FORTRAN subroutines

#### 3.4.1 Subroutine for adiabatic change of gas

As it was mentioned before, the calculations of fluid-structure interaction problem performed by ABAQUS/Standard require assumption of isothermal process or applying prescribed change of temperature in time. This means that the temperature of the gas cannot be calculated during the analysis and the results of temperature change on gas
and structure properties cannot be appropriately taken into account. It is especially important in the case of fast changes of gas (like in case of impact on the container) when gas does not exchange its energy with the surrounding structure and it temperature changes significantly.

This limitation of the ABAQUS/Standard was overcome by using FORTRAN subroutines: URDFIL and UTEMP. For the sake of simplicity the case when no flow between the chambers and outside the structure occurs was considered. We will also assume that constant volume heat capacity \( c_v \) is not temperature dependent. The subroutine URDFIL was used to read the actual values of pressure and volume of the cavity. We will use the equation of energy conservation (7) in the form: \( mc_v \cdot d\theta = -p \cdot dV \). Substituting the differentials by finite differences gives us:

\[
mc_v(\theta_{\text{NEW}} - \theta_{\text{OLD}}) = -p(V_{\text{NEW}} - V_{\text{OLD}})
\]  

(44a)

By writing this equation for the pressure \( p \) equal to average between the time steps we obtain the final formula for calculating new temperature of the gas:

\[
\theta_{\text{NEW}} = \theta_{\text{OLD}} - \frac{(p_{\text{OLD}} + p_{\text{NEW}})(V_{\text{NEW}} - V_{\text{OLD}})}{2mc_v}
\]  

(44b)

Then the calculated temperature was applied on subsequent time step by UTEMP subroutine. The following calculations were performed with new temperature. The scheme of the whole subroutine is the following:

0. Initialization of the data for subroutine: file utemp_out.dat contains data from the last increment of inflation: \( pcav_{\text{old}}, cvol_{\text{old}}, \text{temp}_{\text{old}} \) and file urdfil_out.dat contains data from first increment after inflation: \( pcav_{\text{new}}, cvol_{\text{new}} \)

1. First step: inflating the structure by using pressure boundary condition and assumed change of temperature (the subroutine is not applied here)

2. Second step: removing pressure boundary condition and activation of the subroutine

3. UTEMP subroutine reads the data from the files utemp_out.dat and urdfil_out.dat. New gas temperature is calculated and applied for ABAQUS analysis. New values of pressure, volume and temperature are written to the file utemp_out.dat

4. The increment of the ABAQUS analysis is executed. Temperature 'temp_new' is used to calculate the volume and pressure in the cavity.
5. After the time increment subroutine URDFIL is used to read the actual values of volume and pressure and write them to the file urdfil_out.dat

6. The steps 3 to 5 are repeated until all increments in the step are executed

Comment: Artificial initialisation (step 0) is required since subroutine UTEMP is launched before the first time increment when there are no results written to the files. At this first step, the values pcav_new, cvol_new are usually not known and they can be assumed to be equal pcav_old, cvol_old, respectively.

By using the above subroutine, we model in a simplified way adiabatic change of the gas. The method is approximate because conservation of the energy is not fulfilled on every time step, but between the time steps. In other words, instead of solving Eqs. (1, 2, 4) simultaneously, we solve Eqs. (1, 2) and then having the result we solve Eq. (4). However, when the time discretisation is fine enough the difference in pressure and volume between the time steps is small so the shift of a one time step does not affect the result. The comparison with the fully adiabatic analysis conducted in ABAQUS/Explicit (cf. Sec. 3.4.1) shows very good accuracy of the results (cf. Fig. 8). The expected delay of the temperature value, when using FORTRAN subroutine, is clearly visible.

![Graphs of pressure, volume, and temperature](image)

Figure 8. Change of pressure, volume and temperature calculated by user subroutine in ABAQUS/Standard (red line) and obtained from ABAQUS/Explicit (blue line).

The first, not successful attempt to solve this problem was based on the equation of the adiabatic change of gas: \( pV^\kappa = \text{const.} \), cf. (7c) where \( \kappa = c_p/c_v \) and specific heat \( c_p \) is temperature dependent according to Shomate equation (6a). However, it was found from numerical calculations that Eq. (7c) is not satisfied precisely during the entire transient analysis with reasonable number of time steps, presumably due to the nonlinearities involved. The numerical error is equal to several percent. On the other hand, the influence of temperature on coefficient \( \kappa \) and on value of the expression (7c) is quite small (equal also several percent). Basing on these two facts calculating the
change of temperature from (7c) was not possible due to numerical problems. The method is theoretically correct, however not useful in practice.

The reason of applying adiabatic change of gas in ABAQUS/Standard is that while using this solver we can implement other subroutines to control the fluid and its flow according to the actual results of the analysis. The presented subroutine is valid not only in considered example but in general case. After slight changes this subroutine can be applied for any ABAQUS/Standard analysis.

### 3.4.2 Subroutine for adiabatic flow through orifice

According to Sec. 3.2.3 there are three possibilities of modelling flow in ABAQUS/Standard: using prescribed flux flux, applying pressure boundary condition and using fluid link element. None of these methods is able to simulate the adiabatic flow of the gas through orifice since by default the assumption of isothermal process is implemented in ABAQUS/Standard. This assumption is not well suited for the case of rapid events like airbag deployment or fast outside flow.

The equation which is the most commonly used in the literature for adiabatic flow is Saint-Venant-Wantzel formula. According to SVW equation the mass flow rate through vent or exhaust orifice is equal to:

$$|q| = CA \frac{\overline{p}}{\sqrt{R(\theta - \theta_2)}} \sqrt{\frac{2\kappa}{\kappa - 1} \left(\frac{Q^{2/\kappa}}{Q^{(\kappa+1)/\kappa}} - 1\right)}$$  \hspace{1cm} (45)

where $C$ is discharge coefficient, $A$ is the area of the orifice, $\overline{p}$ is the absolute pressure in the upstream fluid cavity, $Q$ is the pressure ratio defined as $Q = \frac{p}{p_e}$, where the absolute pressure in the orifice $\overline{p}$ is given by:

$$\overline{p} = p_A \text{ if } p_A \geq p_C$$  \hspace{1cm} (46)

$$\overline{p} = p_C \text{ if } p_A < p_C$$

where $p_A$ is the ambient pressure for the flow outside the cavity or pressure in the downstream cavity pressure for the flow between two cavities; $p_C$ is critical pressure at which choked or sonic flow occurs, given by:

$$p_C = p_e \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$$  \hspace{1cm} (47)
Formula (47) is found as an extremum of function (45): $dq/dQ = 0$. Hence, for critical pressure the highest mass flow rate occurs. Value of the critical pressure for the air is equal to: $\rho_c = 0.528 \rho_v$.

Standard equation used for the description of the fluid link based on (4) with hydrodynamic coefficient $C_H = 0$ has the form:

$$q = \Delta p / C_v \tag{48}$$

Implementation of the Saint-Venant-Wantzel formula into ABAQUS/Standard is possible by using FORTRAN subroutines: URDFIL and UFIELD. Initially, the subroutine URDFIL is used to read actual values of pressure, volume and temperature of the gas in the cavity. Then the comparison of the formulae (45) and (48) can be done to calculate the value of $C_v$ in terms of pressure which will properly model adiabatic flow. We obtain:

$$C_v = \frac{\Delta p \sqrt{R(\theta - \theta_z)}}{CA \rho_v \sqrt{\frac{2\kappa}{\kappa - 1}(Q^{2/\kappa} - Q^{(\kappa+1)/\kappa})}} \tag{49}$$

The subroutine UFIELD is used to calculate the value of ABAQUS field variable and implement this field variable to the value of coefficient $C_v$. The subroutine performs the following tasks:

0. Initialization of the data for subroutine: subroutine file svw.for contains gas temperature and desired value of the discharge coefficient $C$; file urdfil_out.dat contains value of the pressure at the last step of the inflation

1. First step: inflating the structure by using pressure boundary condition and assumed change of temperature (the subroutine is not applied here)

2. Second step: removing pressure boundary condition and activation of the subroutine

3. UFIELD subroutine reads the pressure from the files ufield_out.dat. It calculates the value of critical pressure from Eq. (47), absolute pressure in the orifice from condition (46) and finally the value of field variable $C_v$ from (49)

4. The increment of the ABAQUS analysis is executed. Calculated by UFIELD coefficient $C_v$ is used to compute mass flow of the gas
5. When the calculations for one time increment are done, subroutine URDFIL reads the actual pressure in the cavity and writes it to the file urdfil_out.dat.

6. The steps 3 to 5 are repeated until all increments in the step are executed.

The exemplary results obtained from the above subroutine for the case of impact applied to the pressurised structure are presented below in Figures 9a and 9b.

![Figure 9a. Pressure and volume calculated by subroutine (red line) obtained from ABAQUS/Explicit (blue line).](image)

![Figure 9b. Resulting change of coefficient $C_V$ calculated by the subroutine.](image)

The disadvantage of this method is that the calculation of the coefficient $C_V$ is done with the assumption that temperature is constant. Hence, the description of the process is not consequent. For the fully correct modelling of the flow the joint use of subroutines for adiabatic change of gas and for adiabatic flow has to be implemented. All the components on the r.h.s. of the formula (5b) should be taken into account and three subroutines: URDFIL, UTEMP, UFIELD should be used.
3.4.3 Subroutine for defining fluid density and fluid compliance

Subroutine UFLUID can be used for defining user constitutive model of the fluid. When using this subroutine fluid described by equation of state different than ideal gas law can be modelled. The fluid density, fluid pressure compliance and fluid temperature compliance have to be defined inside the subroutine. In case of ideal gas its density expressed as a function of reference and initial state equals to:

\[
\rho = \frac{\rho_R (\theta_R - \theta_Z)(p - p_A)}{(p_R - p_A)(\theta - \theta_Z)} \tag{50}
\]

Fluid pressure compliance is defined as derivative of unit fluid volume over pressure:

\[
C_p = \frac{dv}{dp} = \frac{d(1/\rho)}{dp} = -\frac{1}{\rho^2} \frac{dp}{d} = \frac{-(\theta - \theta_Z)(p_R + p_A)}{\rho_R(\theta_R - \theta_Z)(p + p_A)^2} \tag{51}
\]

Fluid temperature compliance is defined as derivative of unit volume over temperature:

\[
C_T = \frac{dv}{d\theta} = \frac{d(1/\rho)}{d\theta} = -\frac{1}{\rho^2} \frac{d\rho}{d\theta} = \frac{(p_R + p_A)}{\rho_R(\theta_R - \theta_Z)(p + p_A)} \tag{52}
\]

If the relations for considered fluid are different than Eqs. (50–53) then using UFLUID subroutine is necessary. Model of the fluid is fully defined by Equation (50), however, formulas for both fluid compliances must be also given explicitly. UFLUID subroutine can be useful for modelling behaviour of real gas whose equation of state is often obtained by adding function \( Z \) to the equation of the ideal gas law:

\[
\bar{p} = Z \cdot \rho \cdot R \cdot (\theta - \theta_Z) \tag{53}
\]

Function \( Z \) is defined separately for every fluid. The subroutine UFLUID was not implemented into the model so far.
4. Feasibility study of the inflatable structures for protecting wind turbines

4.1 Considered wind turbine tower

![Diagram of wind turbine tower](image)

**Figure 10. Dynamic model of the wind turbine tower (courtesy of A. Mróz [4]).**

The Adaptive Inflatable Structure introduced in Sec. 2 will be used for the purpose of protecting offshore wind turbine (cf. Fig. 10) against collisions of small ships. The pressurised structure has a shape of torus surrounding the tower (cf. Fig. 11) and it is located just above the cone structure on the water level. The weight of the pressurised structure is not taken into account since it was considered to be small enough to be neglected.

It was assumed that the inflatable structure has the height of 2 m and its thickness can vary from 0.5 m to 1 m. The exact parameters of this structure will be found after considering the conditions necessary for optimal impact absorption (cf. Sec. 4.4.1).

- **Blade** (S4R5 shell elements): Material properties and shell thickness were chosen so that the first own frequency and weight of blade are $\omega_1 = 1.30 \text{ Hz}$, $G = 7006 \text{ kg}$

- **Tower** (B33 beam elements): 47 elements with circular section were used, radius $R = 1.161 \div 2.111 \text{ m}$, thickness $t = 0.014 \div 0.040 \text{ m}$. Weight of tower $G_T = 186782 \text{ kg}$

- **Point masses** (MASS elements): $MT_1 = 13000 \text{ kg}$, $MT_2 = 31800 \text{ kg}$ (turbine), $MT_3 = 35000 \text{ kg}$, $MF_1 = 4200 \text{ kg}$, $MF_2 = 3200 \text{ kg}$ (flanges), $MF_3 = 1900 \text{ kg}$, $MF_4 = 2300 \text{ kg}$, $MC = 49620 \text{ kg}$ (cone + water)

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- $MF_1 = 4200 \text{ kg}$
- $MF_2 = 3200 \text{ kg}$ (flanges)
- $MF_3 = 1900 \text{ kg}$
- $MF_4 = 2300 \text{ kg}$
- $MC = 49620 \text{ kg}$ (cone + water)
4.1.1 Parameters of the reduced model

Initially both the wind turbine and the ship were reduced to one and two-dimensional objects on the water level, cf. Fig. 12 and Fig. 18 accordingly. Reduced model cannot describe precisely the response of the tower but it is less computationally expensive and better suited for performing optimization and control.

\[ M = \frac{1}{\phi^2_{1C}} \int_0^H m(z) \cdot \phi_1^2(z) \, dz = \frac{M_{11}}{\phi^2_{1C}} \]  

\[ K = \omega_1^2 M = (2\pi f_1)^2 M \]  

\[ C = 2\xi \omega_1 M \]

where: \( M_{11} \) and \( K_{11} \) are generalised mass and stiffness for the first mode, \( \phi_1 \) is the eigenvector (mode shape), \( \phi_{1C} = \phi_1 (z_C) \) is the value of the first mode on the water level, \( \omega_1 \) is the first circular frequency and \( \xi \) is damping coefficient. Values of the parameters \( f, \phi_{1C}, M_{11} \) can be read directly from ABAQUS analysis of the model from Fig. 10 The resulting mass, stiffness and damping of the reduced model are presented in Table 3. (first two modes are considered).
### Table 3. Parameters of the 1D model on the water level.

<table>
<thead>
<tr>
<th></th>
<th>( f ) [1/s]</th>
<th>( \phi_c ) [m]</th>
<th>( M1 ) [kg]</th>
<th>( M ) [kg]</th>
<th>( K ) [N/m]</th>
<th>( C ) [Ns/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mode 1</strong></td>
<td>0.33147</td>
<td>1.584e-3</td>
<td>116 748</td>
<td>0.4653e11</td>
<td>0.2018e12</td>
<td>0.9690e9</td>
</tr>
<tr>
<td><strong>Mode 2</strong></td>
<td>0.38503</td>
<td>1.021e-3</td>
<td>34 646</td>
<td>0.3318e11</td>
<td>0.1942e12</td>
<td>0.8029e9</td>
</tr>
</tbody>
</table>

The stiffness of the one DOF system is modelled by beam element. The cross-section and length of this element are assumed arbitrarily as 0.01 m² and 4.48 m respectively. Thus, the Young modulus of the beam element was calculated as: \( E = KL/A \) and it was equal to \( E = 9.042 \times 10^{13} \) N/m². Damping of the beam element was assumed to be Rayleigh damping with coefficients \( \alpha = 0.0112, \beta = 0.0022 \), which were calculated using two first eigenmodes frequencies and damping coefficient \( \xi = 5\% \) according to the formula:

\[
\xi = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}
\]  

(55)

To check whether the reduction is done correctly the 1 DOF system was subjected to harmonic loading with the frequency \( \omega_p \) close to a natural frequency of the system \( \omega \) and amplitude \( P_0 \). The analytical results for the displacements of the tower calculated according to:

\[
u(t) = \frac{P_0}{K} \frac{1}{1 - \beta^2} \left( \sin \omega_p t - \beta \sin \omega t \right)
\]  

(56)

where \( \beta = \omega_p / \omega \), were comparable to the displacements obtained from 3D analysis, cf. [5] what indicates that the reduction was performed correctly.

### 4.2 Simplified mass-spring models

In this chapter three simplified models will be introduced: one-spring model, double-spring model, spring-piston model. Modelling collision by using these two models is very rough estimation of the real situation however it provides basic estimation of the system efficiency.

**One spring model**

The simplest possible method of modelling impact of the ship is assuming the elastic collision, cf. Fig 13. The governing equations indicate conservation the impulse (57a) and conservation of the energy (57b):
\[ mV_0 = MV_T + mV_S \quad (57a) \]
\[ 1/2 \cdot mV_0^2 = 1/2 \cdot MV_T^2 + 1/2 \cdot mV_S^2 \quad (57b) \]

where \( V_0 \) is initial ship velocity and \( V_T, V_S \) are unknown tower and ship velocities just after the impact. After solving the system (57) we obtain:
\[ V_T = \frac{2mV_0}{m+M}, \quad V_S = -\frac{(M-m)V_0}{m+M} \]
for the tower and ship velocities, respectively.

![Figure 13. One degree of freedom model of the ship colliding into the wind turbine.]

The amplitude of a harmonic motion of the wind turbine can be found as a solution of the equation of motion and it is equal to:
\[ u_{\text{max}} = V_0 \left( \frac{K}{M} \right)^{-1/2} = \frac{2mVM^{1/2}}{(m+M)K^{1/2}} \quad (58a) \]

In our case, when the mass of the ship is many times smaller than the reduced mass of the wind turbine \( m \ll M \) the above formula is simplified to:
\[ u_{\text{max}} = \frac{2mV}{M^{1/2}K^{1/2}} \quad (58b) \]

Displacement of the tower top can be calculated by dividing this value by the displacement on the water level in the eigenmode normalised to the displacement of the tower top: \( V_{\text{top}}^{\text{top}} = V_T / \phi_C \) and \( u_{\text{max}}^{\text{top}} = u_{\text{max}} / \phi_C \). For the strongest considered impact of the 80-tons ship with the velocity of 8m/s values of initial velocity of the tower top and its displacement are equal: \( V_{\text{top}}^{\text{top}} = 0.0173m/s \), \( u_{\text{max}}^{\text{top}} = 0.008329m \). Equation (58b) indicates that maximal tower displacement depends on the impulse of the ship, not on its energy.

**Double-spring model**

The other simplified approach to the problem is assuming a two-mass, two-spring system, cf. Fig. 14. The left spring is modelling stiffness of the tower and the right one the stiffness of the inflatable structure. The system of equations can be significantly...
simplified by assuming linearity of the second spring. This assumption does not describe properly properties of the inflatable structure, however it helps to estimate the range of influence of the spring stiffness on the solution. Additionally, \( u_1 \) indicates displacement of the tower and \( u_2 \) indicates displacement of the ship.

![Figure 14. Two degree of freedom model of the ship colliding into the wind turbine.](image)

When composing governing equations of the system, one should take into account that the right spring acts only during compression. Finally we obtain:

\[
M \frac{d^2 u_1(t)}{dt^2} + K u_1(t) - \begin{cases} k(u_2(t) - u_1(t)) & \text{for } u_2(t) < u_1(t) \\ 0 & \text{otherwise} \end{cases} = 0
\]

\[
m \frac{d^2 u_2(t)}{dt^2} + \begin{cases} k(u_2(t) - u_1(t)) & \text{for } u_2(t) < u_1(t) \\ 0 & \text{otherwise} \end{cases} = 0
\]

\[IC : u_2(0) = -V_0\]

The response of the system can be divided into two phases. In the first one (just after the impact) both springs are acting and the solution is superposition of two eigenmodes. In the second phase, mass M is performing free vibrations. However, the mentioned analytic solution is quite complicated and alternatively the system can be solved numerically.

When considering the impact of the 80-tons ship with the initial velocity of 8 m/s and assuming various values of stiffness of a spring \( k \), it was found that the displacements of the tower are significantly smaller than displacements of the ship (approx. more than 100x). Taking into account the above fact and the fact that stiffness \( k \) is significantly smaller than stiffness \( K \) we can neglect the term \( ku_1(t) \) in both equations. Therefore, a set of equations (59a) changes to:

\[
M \frac{d^2 u_1(t)}{dt^2} + K u_1(t) - \begin{cases} k u_2(t) & \text{for } u_2(t) < u_1(t) \\ 0 & \text{otherwise} \end{cases} = 0
\]

\[
m \frac{d^2 u_2(t)}{dt^2} + \begin{cases} k u_2(t) & \text{for } u_2(t) < u_1(t) \\ 0 & \text{otherwise} \end{cases} = 0
\]

\[IC : u_2(0) = V_0\]
We immediately notice that the set of equations was separated and they can be solved much more easily. The solution of the second equation for the first phase of impact \((u_2(t) < u_1(t))\) equals:

\[
u'_2(t) = \frac{V_0}{\sqrt{k/m}} \cdot \sin(\sqrt{k/m} \cdot t) \tag{60a}
\]

By introducing this value to the first equation we obtain:

\[
u'_1(t) = \frac{kV_0}{K \sqrt{k/m}} \cdot \frac{1}{1 - \beta^2} \left( \sin(\sqrt{k/m} \cdot t) - \beta \sin(\sqrt{K/M} \cdot t) \right) \tag{60b}
\]

and coefficient \(\beta\) is defined as: \(\beta = \omega_2 / \omega_1\), \(\omega_2 = \sqrt{k/m}\), \(\omega_1 = \sqrt{K/M}\). In the second phase, when free vibrations of the tower occur \((t > t^* = \pi / \sqrt{k/m})\) we obtain, respectively:

\[
u''_2(t) = V_0 \cdot (t - t^*) \tag{61a}
\]

\[
u''_1(t) = \frac{u'_1(t^*)}{\sqrt{k/m}} \cdot \sin(\sqrt{k/m} \cdot (t - t^*)) - u'_1(t^*) \sin(\sqrt{K/M} \cdot (t - t^*)) \tag{61b}
\]

Similarly, as in a one-spring example, displacement of the tower top can be calculated as: \(u'^{\text{top}}_1 = u'_1 / \phi_C\) and \(u''^{\text{top}}_1 = u''_1 / \phi_C\).

Let us consider the optimal parameters of the right spring which models the inflatable structure. Requirements for the spring are that it provides smallest forces (and thus smaller accelerations) acting on the ship and smallest displacement of the wind turbine tower. The restriction for the spring parameters is that no collision between masses is allowed which means that maximal ship displacement is smaller than the spring length. This condition reads, as follows:

\[
u_2^\text{max} = \frac{V_0}{\sqrt{k/m}} \leq l_0 \Rightarrow k \geq \frac{mV_0^2}{l_0^2} \tag{62}
\]

Minimisation of maximal ship accelerations can be written as:

\[
u_\text{min} = \min_k \max_t u(t) = \min_k \max_t V_0 \omega_2 \sin(\omega_2 t) = \min_k V_0 \sqrt{k/m} \tag{63}
\]

The solution to problem (63) is the smallest \(k\) admissible which equals \(k_{\text{opt}} = mV_0^2 / l_0^2\). Hence, optimal spring stiffness equals to its critical stiffness which stops the ship just before the wind turbine. At this moment maximal force acting on the ship equals:
We conclude that the maximal force acting on the ship is proportional to its energy and inversely proportional to the spring length. In the case of 80-tons ship with the initial velocity of 8 m/s and by assuming spring length of 0.7m we obtain acceleration of \( \dot{u}^\text{max}_2 = 91.43 \text{ m/s}^2 \) and force \( P^\text{max} = 7.31 \text{MN} \). The time of impact can be calculated as:

\[
t = \pi / \sqrt{k_\text{opt} / m} = \pi l / V_0 = 0.275 \text{s}.
\]

The second condition, that optimal spring should satisfy, is minimal displacement of the wind turbine tower. Maximal displacement of the tower in the first phase can be found from the condition that the tower velocity equals zero, which yields:

\[
t^\text{max} = 2 \pi (\omega_p + \omega),
\]

where \( \omega_p \) is frequency of the loading and \( \omega \) is the eigen frequency of the tower. Tower displacement equals:

\[
u^1_{\text{max}}(t_{\text{max}}) = \frac{kV_0}{K\omega_p} \left( \frac{1}{1 - \beta^2} \left( \sin \left( \frac{2\pi}{1 - \frac{1}{\beta^2}} \right) - \beta \sin \left( \frac{2\pi}{\beta + 1} \right) \right) \right)
\]

From the comparison of time \( t_{\text{max}} \) and time of the first phase \( t^* \) we conclude that the solution is valid for \( \omega_p < \omega \) which is equivalent to: \( k_2 < k_1 m/M \). Maximal tower displacement in the second phase can be calculated as the amplitude of the function \( u^2_i(t) \) and it equals simply:

\[
u^2_{\text{max}} = -\frac{kV_0}{K\omega_p} \frac{2\beta}{1 - \beta^2} \cos \frac{\pi}{2\beta}
\]

The displacement of the tower top was calculated for various stiffness of inflatable structure, cf. Fig. 15. The length of the spring was assumed \( l = 0.7 \text{ m} \) and minimal stiffness \( k_{\text{min}} = 210 e 3 \text{ N/m} \).

In Fig. 15 red line indicates the first phase of impact and the blue line the second phase. The conclusion is that even huge decrease the inflatable structure stiffness (up to 5 times in the considered example) changes response of the tower insignificantly. General conclusions from the above simplified analysis are that by using the inflatable structure of the reasonable dimensions we are unfortunately not able to mitigate the vibrations of the wind turbine tower. This is mainly due to the fact that the frequency of the tower vibrations is significantly smaller than the frequency of the ship impulse. Our conclusion is in agreement with the fact that for short impacts the response of the system does not depend on the shape of excitation, but mainly on the integral

\[
I = \int P(t) dt, \text{ cf. [1].}
\]

In our case this integral equals:
\[ I = \int_0^\pi \frac{kV_0}{\omega_p} \sin(\omega_p t) dt = -\frac{kV_0}{w_p} \cos(\omega_p t) \bigg|_0^\pi = 2V_0m \]  

which confirms that the response of the tower depends on the ship impulse.

\[ 0 \leq t \leq 2 \]

Figure 15. Tower top displacement for various inflatable structure stiffness: a) \( k = 210e3 \text{ N/m} \), b) \( k = 500e3 \text{ N/m} \), c) \( k = 1000e3 \text{ N/m} \).

The method of using pressurised structures can still be useful for different types of structures whose natural frequency is higher than for wind turbine tower. Examples of such structures are for instance bridge piers. The feasibility study was done for the structures whose stiffness is 10 and 100 times larger than of the wind turbine tower, which effects in 3.33 and 10 times increase of natural frequency. Maximal tower displacement and time when it is attained were calculated in terms of wide range of the inflatable structure stiffness, cf. Fig. 16.

Figure 16. Change of maximal tower displacement (a) and time when it is attained (b) in terms of inflatable structure stiffness for various tower stiffness (K, 10K, 100K).

The results are normalised with respect to maximal tower displacement which occurs for largest inflatable structure stiffness. We can also observe whether maximal tower displacement occurs in the first or second phase of impact. Presented results indicate
clearly that the response of the tower can be influenced significantly by changing the inflatable structure stiffness.

The above plots are slightly similar to well known in structural dynamics “response-spectra” which are used to identify maximal response of one d.o.f system. The difference is that here we analyse the response of double mass, double spring system, not one degree of freedom model. The response is presented in terms of stiffness not in terms of frequency since the inflatable structure stiffness influences both amplitude and frequency of the force acting on the tower. One should remember that the above results concern the simplified case described by Equations (59b) where the assumption of large stiffness $K$ with comparison to $k$ was done. In general case the Equations (59a) instead of (59b) should be used.

**Spring-piston model**

Another method of simplified modelling of the ship colliding into tower protected by inflatable structure is presented in Fig. 17. The linear spring is now replaced with the air spring described in Sec. 3.2.

![Diagram of two degree of freedom model of the ship colliding into the wind turbine.](image)

**Figure 17. Two degree of freedom model of the ship colliding into the wind turbine.**

When the pressure inside the chamber is arbitrarily assumed, the equations governing the problem are, as follows:

\[
\begin{align*}
M \frac{d^2 u_1(t)}{dt^2} + K u_1(t) + \begin{cases} (\bar{p}(t) - p_a)A & \text{for } u_2(t) < u_1(t) \\ 0 & \text{otherwise} \end{cases} &= 0 \\
\frac{m}{A} \frac{d^2 u_2}{dt^2} - \begin{cases} (\bar{p}(t) - p_a)A & \text{for } u_2(t) < u_1(t) \\ 0 & \text{otherwise} \end{cases} &= 0
\end{align*}
\]

(67)

The above formulation is very general, and we can rearrange it by assuming pressure as a function of other parameters. By using the standard definition of pressure in terms of initial pressure, piston displacement, mass added to the system and external temperature (cf. 9b) we can express the force acting in the air spring as:
\[ P_{\text{air}}(t) = (p(t) - p_A)A = \left( \frac{h_0 p_0 + \Delta m(t) \cdot R}{\partial_0} \cdot \frac{\bar{\theta}(t)}{h_0 - (u_1(t) - u_2(t))} - p_A \right)A \]  

By introducing an appropriate function of temperature, cf. (30b) we can obtain the equations for isothermal or adiabatic process. In the case when we consider flow of the gas through the valve the equation (68a) changes to:

\[ \bar{p}(t) = \left( \frac{h_0 p_0}{\partial_0} + \int_0^t \frac{\bar{p}(t) - p_A}{C_1(t)} \cdot \frac{\bar{\theta}(t)}{h_0 - (u_1(t) - u_2(t))} \right) dt \]  

The last formula can be easily transformed into differential equation as in Sec. 3.2. Finally, the set of equations governing the problem is composed of three differential equations: two linear equations of motion for both masses (67) and a nonlinear relation between the pressure inside the air spring and piston displacement (68b). Due to the fact that in the considered problem tower displacements are relatively small in comparison to ship displacements we can neglect the term \( u_1(t) \) in Eqs. (68a) and (68b). By applying this simplification and assuming isothermal process without change of mass we obtain:

\begin{align*}
M \frac{d^2u_1(t)}{dt^2} + Ku_1(t) + \left\{ \begin{cases} 
\left( \frac{h_0 p_0}{h_0 + u_2(t)} - p_A \right)A & \text{for } u_2(t) < u_1(t) \\
0 & \text{otherwise}
\end{cases} \right. = 0 \\
m \frac{d^2u_2}{dt^2} - \left\{ \begin{cases} 
\left( \frac{h_0 p_0}{h_0 + u_2(t)} - p_A \right)A & \text{for } u_2(t) < u_1(t) \\
0 & \text{otherwise}
\end{cases} \right. = 0
\end{align*}

These equations are coupled, however we can solve them separately. We can initially solve the second, nonlinear equation and then solve the first, linear equation treating its last term as time dependent excitation.

For the purpose of optimal adjustment of the air spring characteristics we will use (67). The elementary difference between this and the previous approach is that now we can change pressure during the whole analysis. In contrast, in the previous case the spring characteristics were constant during whole impact.

The condition that the ship must be stopped before hitting the wind turbine tower can be obtained from the energy conservation and it reads:

\[ \int_{0}^{u_{\text{stop}}} - A(p(t) - p_A)du \geq \frac{mV_0^2}{2} \]  

60
Let us consider the minimisation of forces acting on the ship. The optimisation problem can be formulated as:

$$ u_{\text{min}} = \min_{p(t)} \max_{\|u(t)\|} \frac{A(p(t) - p_A)}{m} = \min_{p} \frac{A(p - p_A)}{m} \quad (71a) $$

Therefore, the optimal pressure is constant and equal to minimal admissible pressure according to (70). By introducing maximum piston displacement $u^{\text{stop}} = -h_0$ and constant pressure into (71a) we obtain the following results:

$$ p_{\text{opt}} = \frac{mV_0^2}{2Ah_0} + p_A, \quad P_{\text{max}} = \frac{mV_0^2}{2h_0}, \quad u_{\text{opt}}^2(t) = \frac{V_0t^2}{4h_0} + V_0t \quad (71b) $$

The conclusion is that the value of optimal pressure in the chamber depends on the ship energy and it is inversely proportional to the area of the piston and its length. When using the same length of the piston as the length of the linear spring considered previously the forces and accelerations are decreased exactly two times and they are constant during the whole analysis and equal: $\ddot{u}^{\text{max}}_2 = 45.71 \text{ m/s}^2, \quad P^{\text{max}} = 3.65MN$. The time of impact can be calculated as: $t = 4h_0/V_0 = 0.35s$.

Having optimal value of pressure (71b) and displacement of the piston we can calculate the necessary increase of the mass of the gas in the chamber from (68a) with the displacement $u_1(t)$ neglected. We can also calculate the flow coefficient $C_v(t)$ according to (68b).

We will also consider minimisation of ship accelerations in case when the initial pressure is equal to atmospheric pressure $p_A$. The optimal strategy is that we keep the valve closed to reach the optimal pressure and then we keep the pressure on the constant level. In the case of isothermal process the equation describing the motion of the ship becomes:

$$ m\frac{d^2u_2}{dt^2} = \begin{cases} \left( \frac{h_0p_0}{h_0 + u_2(t)} - p_A \right) A & \text{for} \quad t < t^* \\ \left( \frac{h_0p_0}{h_0 + u_2(t^*)} - p_A \right) A & \text{otherwise} \end{cases} = 0 \quad (72) $$

Where $t^*$ is the time when the valve must be opened. Due to the fact that the work done by gas must equal the initial energy of the ship we can calculate the displacement $u_2(t^*)$

$$ \frac{mV_0^2}{2} = h_0p_A \ln \left( \frac{h_0}{h_0 + u_2(t^*)} \right) + h_0p_0A(h_0 + u_2(t^*)) - p_AAh_0 \quad (73) $$
The time $t^*$ of valve opening can be computed using the Equation (72). In this case the value of forces and accelerations of the ship are higher then in case when we are able to keep pressure constant during whole impact.

The optimisation of pressure oriented towards minimisation of the tower vibrations has given similar results as in the case of linear spring. Various pressure impulses satisfying Equation (70) were considered. Maximal displacement did not depend significantly on the shape of impulse. The impulse which causes the smallest tower displacement was the one of the longest duration and constant pressure.

4.3 Modelling collision with 2D off-shore structure

For the purpose of more precise modelling of the impact subjected to offshore wind turbine and its cooperation with the inflatable structure, two-dimensional model one the water level was implemented, cf. Figs. 2 and 18. The analysed structure was the wind turbine introduced in Sec. 4.1 and reduced to the water level as in Sec. 4.1.1.

The model

The whole model was done as MATLAB script which creates input file for the ABAQUS program. The reason for choosing this approach was the possibility of easy parameterisation of the model and the fact that CAE preprocessor does not support the capability of fluid-filled cavities. The parameters of the model were the following:

- geometry of the model (radius of the tower, radius of AIS, number of chambers in AIS, number of structure stiffeners)
- material properties of the tower and of the AIS
- mass, stiffness, damping of the reduced model
- properties of the gas and its initial conditions
- properties of the gas flow
- mass, velocity, length and distance of the ship
- number of the nodes, length of both time steps, length of the single increment.

Executing multiple runs of ABAQUS program was easily obtained by applying loops in MATLAB code. This approach ensures an easy future implementation of MATLAB optimisation procedures.

Structural part

The analysed structure was made of beam elements, cf. Fig. 18. The type of elements used was B21: linear Timoshenko beams since they are recommended for the impact
applications. The stiffness of the tower was modelled by additional element connected in its middle point. Thus additional elements connecting tower with the walls of the tower were required. Linear material model was applied both for the tower and for the rubber since the deformations of the rubber were large but the strains were quite small. The mesh was finer in the front part of the pressurised structure where impact was applied.

Additional mass obtained from a reduction of the full model (according to Table 3) was located in the middle of the structure. A disadvantage of this approach is that the rotation of the whole structure during the strong impact may appear. To avoid this problem the rotational degree of freedom in the central node was blocked. An alternative approach is artificial increase of the whole structure material density. In this case the elastic wave velocity decreases and computational time is reduced significantly, however, the resulting deformation and internal forces in the structure are different.

The stiffness of the tower on the water level is so huge that the displacements of its middle point are several times smaller than the displacement of the rubber elements. The tower displacement is in this case strongly affected by numerical error. To avoid the problem mass and stiffness of the tower were scaled by the ratio of the displacement on the water level to the displacement of the tower top \( \phi_1 \). This way the resulting displacement refers to the tower top. The procedure is valid only for the simplified system described by Equation (69) due to the linearity of Equation (69a) in terms of tower displacement \( u_1(t) \).

Additionally, height of the considered 2D model is reduced in comparison to 3D inflatable structure so we have to decrease the mass of the tower, mass of the ship and tower stiffness to provide reasonable deformation of the rubber beams. Mathematically it means dividing both Equations (69) by a constant factor which can be done both for initial and simplified set of equations (67 and 69). Finally, the parameters of the system are changed in the following way:

\[
\begin{align*}
M^* &= M \cdot \phi_{1C} \cdot \frac{h^{2D}}{h^{3D}} \\
K^* &= K \cdot \phi_{1C} \cdot \frac{h^{2D}}{h^{3D}} \\
m^* &= m \cdot \frac{h^{2D}}{h^{3D}}
\end{align*}
\]  

Having taken these changes into account the values of the gas pressure and the displacements are real values and they do not need to be scaled. To check the correctness of these modifications the modal analysis was performed for a 2D model. The value of the first natural frequency was found to be the same as for a three-dimensional wind turbine.
Gas properties and gas flow

The gas inflating the cavities was modelled by “fluid-filled cavities” and “surface-based cavities”. Reference density of the gas under pressure $\bar{\rho}_g = 100e3N/m^2$ and in the reference temperature of 20K was assumed to be 1.1857 $kg/m^3$ (which is typical value for the air). This data gives the value of the gas constant $R$ equal to $R = 287.696 J/kg/K$. Alternatively, in ABAQUS Explicit molecular weight was equal 0.0289kg/mol which gives the same value of the gas constant. Initial temperature was usually equal to 20C and was assumed to be constant or to change according to adiabatic process. Initial pressure varied from 0 to several atmospheres.

The inflation of the cavities was executed during the first step of the analysis. The length of this step and value of pressure was adjusted. The possible directions of the gas flow are the following: from the central (right) cavity outside, from central cavity to top and bottom cavities, from the inside chamber with higher pressure to central cavity. The flow is most often described as in (4b) by viscous and hydrodynamic coefficients.

Loading and solution

Initially, the loading was applied as a mass distributed over the surface of the AIS with initial velocity. This simplified method has two main disadvantages:

- rubber wall deforms in a non-realistic way, cf. Fig. 18a (middle part of the wall exhibits the largest deformation)
- in the second part of the impact, after the mass is bounced from the structure it is pulling the structure to the right which does not occur in the reality.

![Figure 18a. Collision modeled by mass with initial velocity: initial state and resulting deformation.](image)

The more precise method of modelling impact is contact formulation. The ship was modelled as rigid surface having prescribed mass and length and approaching in the
direction of the tower with initial velocity, cf. Fig. 18b. Two contact pairs were defined: between the ship and the rubber wall (with the ship being master surface) and between the rubber wall and the wall of the tower (with the tower wall being master surface). This approach provides a correct deformation of the structure. After the collision part of the AIS wall has common deformation with the ship. During this stage the pressure increases, the ship is stopped and it bounces from the inflatable structure. When impact energy is high the pneumatic structure is not able to stop the ship and contact between the rubber AIS wall and tower wall of the tower occurs. All further results are obtained by using the above method.

The range of possible impacts is: \( m = 20\text{–}80 \) tons, \( V = 2\text{–}8 \) m/s. The resulting impact energies vary from 0.04 MJ to 2.56 MJ. The difference between weakest and strongest impact is 64 times.

![Image](image.png)

*Figure 18b. Collision modeled by contact: initial state and resulting deformation.*

The solution is done by general dynamic step (*DYNAMIC). Large deformations of the rubber part of the structure are considered so the NLGEOM option indicating geometrical nonlinearity is enabled. Automatic time incrementation in ABAQUS/Standard is used by applying HAFTOL option. To provide convergence of the solution the time increment is very short when the contact occurs and when pressure boundary condition is removed. The entire solution takes about several minutes and it is usually shorter when using ABAQUS/Explicit.

### 4.4 Design and adaptivity of the pressurised structure

The inflatable structure surrounding the tower must be designed to fulfill the following goals in case of ship impact:

1. Stop the ship before hitting the tower wall.
2. Dissipate the impact energy.
3. Minimise the accelerations and forces acting on the ship.
4. Mitigate the local internal forces acting on the tower.
5. Decrease amplitudes of the tower vibrations.

All these goals are achieved when the ship is stopped by compressing largest volume of the pressurised structure, its velocity decreases to zero in the vicinity of the tower wall, the pressure inside is kept on the lowest possible level during entire impact and it is released when the ship is stopped.

### 4.4.1 Adjustment of structure parameters

Before adjustment of pressure values and flow coefficients between the packages appropriate values of AIS properties have to be designed. The reasonable approach is to adjust general parameters of the inflatable structure according to particular wind-turbine and initial value and release of pressure according to particular impact. The considered general properties of the inflatable structure were the following:

- number of chambers in the pressurised structure (3, 6, 9, 12)
- width of the pressurised structure (0.5 m, 0.7 m, 1 m)
- the Young modulus and thickness of the AIS wall (5–200 Mpa, 0.5–5 cm).

Indications for a proper choice of these parameters are:

1. The capability of stopping the ship depends strongly on work that can be done by compressed gas: \( W = -\int_0^{\max} (u - u^0) P(u) \, du \). Hence, the average pressure necessary for stopping the ship is inversely proportional to AIS width. By using wide chambers (large \( r_2 \)) we can decrease value of necessary pressure.

2. AIS should resist low and moderate impacts without additional inflating, i.e. with the initial pressure inside the chambers equal to atmospheric pressure. The increase of pressure after full crushing of the chamber most exposed to impact equals:

\[
\bar{p}_{\max} = \bar{p}_0 \frac{V_0 f_0}{V_{\min} f_1} = \bar{p}_0 \cdot W \quad (75)
\]

where: \( V_0 \) is the initial chamber volume given as:

\[
V_0 = \frac{\alpha}{2} \left( r_2^2 - r_1^2 \right) \quad (76)
\]
with $\alpha$ being the chamber length expressed in radians; $V_{\text{min}}$ is the theoretical minimal volume of the chamber (volume calculated at the state when ship hits the tower) given by:

$$V_{\text{min}} = r_1^2 \left( \frac{\alpha}{2} - \frac{\alpha}{2} \right) \text{ for } \alpha \leq \beta = \arccos \left( \frac{r_1}{r_2} \right)$$

$$(77a)$$

$$V_{\text{min}} = r_1^2 \left[ \frac{(r_2^2 - r_1^2)^{1/2}}{r_1} - a \cos \left( \frac{r_1}{r_2} \right) \right] + \left[ \alpha - 2a \cos \left( \frac{r_1}{r_2} \right) \right] \left( r_2^2 - r_1^2 \right) \text{ for } \alpha > \beta$$

$$(77b)$$

$f_0 (\alpha, r_2, p_0)$ is the coefficient describing the increase of the chamber volume at the initial state and $f_1 (\alpha, r_2, p_0)$ is the coefficient increasing the theoretical minimal volume of the chamber. In general $W \sim r_2/\alpha$ which indicates that wide (large $r_2$) and short (small alpha) chambers have a large ratio of $\bar{p}_{\text{max}}$ to $\bar{p}_0$. Since the “ship stopping” capability depends strongly on $\bar{p}_{\text{max}}$, short and wide chambers can absorb stronger impacts with the initial atmospheric pressure.

3. Local stresses in the tower wall depend mainly on the maximal value of pressure. Therefore, for the optimal reduction of the local forces using the longer chambers is more beneficial than the short ones.

4. The mitigation of the vibration during the impact loading can be achieved by extending impact duration above the 0.8T, where T is the period of tower vibrations, cf. [1]. Impulse duration equal to the time of crushing AIS chamber can be calculated with the assumption of constant deceleration of the ship as: $t = 4(r_2 - r_1)/V_0$. This indicates that assuming large value of the radius $r_2$ is beneficial. However, the average impact time equals 0.56s (for average impact and AIS thicknesses) and it is too short to help mitigate vibrations. The decrease of the tower vibrations can also be obtained by decreasing the overall force transmitted to the structure: $P(t) = 2p \sin(\alpha/2)$.

5. When we are using more chambers each of them can be more precisely adjusted to the actual loading conditions.

6. Internal forces in AIS depend on the geometry of the chamber. In long and narrow chambers large stresses appear after initial inflating, in the outer wall. On the contrary, in short and wide chambers large stresses appear in the AIS partitions after crushing of the chamber. We have to ensure the possibility of a large deformation during impact absorption but, on the other hand, we cannot afford any large deformations at the initial state. Both these functions can be fulfilled by using rubber reinforced by steel fibres.
By taking into account all mentioned conditions, especially: $p_0 = 1 \text{ atm} = 101360 \text{ Pa}$, $p_{\text{max}} = 20 \text{ atm}$, maximal stresses in reinforced rubber 70 MPa, maximal initial increase in chamber volume 40%; it was decided to assume the following properties of AIS:

- number of chambers around the tower: 9
- width of the chamber: 0.7 m
- the average Young modulus of AIS wall: 150 MPa
- thickness of AIS wall: 1 cm.

### 4.4.2 Adjustment of pressure: overview

We will analyse various impact applied to 2D model and several exemplary schemes of pressure change to prove the correctness of the conclusions deducted by using simplified mass-spring model. The obtained results will be the guidelines for AIS deployment and controlled release of pressure. Problem of proper pressure adjustment necessary to fulfill the objectives mentioned at the begging of the Sec. 4.4 will be considered precisely in Sec. 4.5 and 4.6.

Initially the tower protected by pneumatic structure was subjected to impact of the same energy of 0.64 MJ but of various velocities, cf. Table 4. Pressure inside the packages is equal to: 0.08 MPa in front cell (the most exposed to impact), 0.05 MPa in adjacent cells and 0.02 hPa in the other cells. We assume that there is no flow between and outside the chambers so the amount of gas remains constant. The calculations are done with the assumption of isothermal process. In all cases the penetration of the ship into the inflatable structure was nearly the same (about 0.5 meter). Therefore, maximal pressure obtained at the moment of stopping the ship was simmilar. The observed values were the following: maximal acceleration acting on the ship, maximal stresses in the tower and displacement of the tower top.

#### Table 4. Response of the structure to the impact of the same energy but various mass and velocity.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20.00</td>
<td>8</td>
<td>0.64</td>
<td>160</td>
<td>80 e3</td>
<td>387.9e3</td>
<td>124.6</td>
<td>140.1</td>
<td>0.00191</td>
</tr>
<tr>
<td>35.56</td>
<td>6</td>
<td>0.64</td>
<td>213.36</td>
<td>80 e3</td>
<td>391.6e3</td>
<td>69.33</td>
<td>133.8</td>
<td>0.00255</td>
</tr>
<tr>
<td>80.00</td>
<td>4</td>
<td>0.64</td>
<td>320</td>
<td>80 e3</td>
<td>392.6e3</td>
<td>31.3</td>
<td>135.9</td>
<td>0.00379</td>
</tr>
</tbody>
</table>

The next problem considered was computing initial pressure (applied only in the main cell) necessary to stop the ship just before the tower wall. Various impact energies were
taken into account. The values of the maximal pressure, displacement of the tower top and maximal stresses in the wall were observed. A comparison with the case when the ship hits the structure directly (without the protection of inflatable structure) was performed. Detailed results are presented in Table 5. The first example concerns impact which can be absorbed by using atmospheric pressure in the chamber. This impact has the energy 0.756 MJ which constitutes 29.5% of the maximal energy considered.

Table 5. Response of the structure for various impact energies and comparison with the structure without AIS.

<table>
<thead>
<tr>
<th>Mass [ton]</th>
<th>Velocity [m/s]</th>
<th>Energy [kJ]</th>
<th>Impulse [t*m/s]</th>
<th>Init. 'p' [N/m²]</th>
<th>Max. 'p' [N/m²]</th>
<th>Stress [MPa]</th>
<th>Displ. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.00</td>
<td>6</td>
<td>756</td>
<td>252</td>
<td>0</td>
<td>609 e3</td>
<td>193</td>
<td>0.00299</td>
</tr>
<tr>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>67.00</td>
<td>6</td>
<td>1206</td>
<td>402</td>
<td>45 e3</td>
<td>1067 e3</td>
<td>340</td>
<td>0.00481</td>
</tr>
<tr>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>52.00</td>
<td>8</td>
<td>1664</td>
<td>416</td>
<td>95 e3</td>
<td>1404 e3</td>
<td>470</td>
<td>0.00499</td>
</tr>
<tr>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>66.00</td>
<td>8</td>
<td>2112</td>
<td>528</td>
<td>150 e3</td>
<td>1711 e3</td>
<td>611</td>
<td>0.00635</td>
</tr>
<tr>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80.00</td>
<td>8</td>
<td>2560</td>
<td>640.00</td>
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<td>1951 e3</td>
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</tr>
<tr>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td>no AIS</td>
<td>no AIS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusions from the conducted analysis:

1. It was found that the pressure change in the chamber fulfils impulse and energy conservation in the form:
   \[ M \cdot V^2_0 \approx \int_{t_1}^{t_2} \Delta p(t) dt, \quad \frac{1}{2} M \cdot V^2_0 \approx \int_0^{\text{max}} \Delta p(\Delta u) du + E^{\text{AIS}}. \]
   The last component indicates the increase of strain energy of the rubber constituting AIS wall. These relations are not fulfilled precisely due to the fact that the energy and impulse of the tower are neglected. The initial pressure, for which whole AIS package is crushed, is proportional to the impact energy. This comes from the fact that the shape of pressure impulse in terms of displacement is nearly the same in all cases.

2. The response of the system depends on the excitation in the following way:
   - Maximal acceleration of the ship is increasing nonlinearly with the ship velocity and decreasing with the ship mass (due to bigger penetration). Hence the lightest ship hitting with high velocities is most exposed to high accelerations.
   - Maximal stress in tower wall is proportional to the highest pressure in the main AIS chamber which depends nonlinearly on impact energy.
   - Displacement of the tower top depends linearly on impulse of the ship and it is relatively very small.

69
3. By using inflatable structure we are able to decrease the accelerations acting on the ship and stresses arising in the tower wall significantly. The reduction of stresses obtained in the above examples was up to 10 times (assuming linear strain-stress relation). The displacement of the tower top was practically the same regardless whether the inflatable structure was used or not. It is due to the fact that the response of the whole tower is strongly determined by loading and by mass and stiffness of the tower.

4.5 Mitigation of ship response

4.5.1 Dissipation of impact energy

In this section our purpose will be to obtain the highest possible dissipation of energy. Due to the fact that after collision most of the energy is accumulated as kinetic energy of the ship, the loss of ship velocity is a good measure of dissipation. By minimising the final ship velocity we will avoid bouncing the ship from the wind turbine.

Energy change during impact is the following: initially the ship has kinetic energy and additionally some energy is accumulated in deformed walls of AIS. During impact kinetic energy of the ship is changed into gas energy, energy of the tower and strain energy of strongly deformed AIS walls, according to (78)

\[ \Delta E_{\text{ship}} = \Delta E_{\text{gas}} + \Delta E_{\text{tower}} + \Delta E_{\text{AIS}} \]  

(78)

At the moment when the ship is stopped its energy \( \Delta E_{\text{ship}} \) equals to zero, and the energies \( \Delta E_{\text{gas}}, \Delta E_{\text{AIS}} \) achieve their maximum values. Strain energy of AIS cannot be neglected here due to the high value of vicarious Young modulus of composite material made of rubber reinforced by steel rods. In the following time of impact major part of the energy accumulated in gas – \( \Delta E_{\text{gas}} \) and in AIS – \( \Delta E_{\text{AIS}} \) is transferred back to the ship. To avoid such phenomenon we dissipate those energies at the moment when gas and AIS energies approach their maximal values.

The dissipation can be obtained in two manners: by removing compressed gas from the pressurised structure to environment and by releasing stresses in AIS walls. The first method is executed by opening the valves in pressurised structure wall. The whole gas energy is dissipated when its pressure equals to atmospheric pressure, since it has no capability of expansion. The second method is performed by changing the stiffness of front AIS partitions which helps to reduce significantly high strain energy accumulated in tensioned outer AIS wall. The above changes are applied according to the formulae:
In the considered examples we will assume that there is no dissipation of the gas energy in the initial stage of impact. Hence, the gas pressure is increasing when the ship is crushing the AIS. We will also assume that the initial pressure is chosen in a way that the ship stops just before the tower wall and then the dissipation of the energy occurs. In numerical analysis we will use FORTRAN subroutine URDFIL to read the actual velocity of the ship during the analysis and FORTRAN subroutine UFIELD to change the area of the valve and change the stiffness of the front AIS partitions. The FORTRAN subroutine scheme is presented below:

0. Initialisation of the data: file ufield_out.dat contains initial values of the field variables indicating closed valve and full stiffness of the AIS partitions; file urdfil_out.dat contains initial velocity of the ship

1. First step: inflating the structure by using pressure boundary condition and assumed change of temperature (the subroutine is not applied here)

2. Second step: removing pressure boundary condition and activation of the subroutine

3. UFIELD subroutine reads the velocity of the ship from the file urdfil_out.dat. As long as ship velocity is positive the field variables indicate closed valve and AIS partitions remains stiff. Field variables are written to ufield_out.dat

4. The increment of the ABAQUS analysis is executed with the valve coefficient and AIS partition stiffness applied by UFIELD subroutine

5. When the velocity of the ship approaches zero the UFIELD subroutine changes the field variables to open the valve and change AIS partitions stiffness

6. The analysis is continued with the AIS parameters changed; ship velocity is still written to urdfil_out.dat to observe the change of ship kinetic energy

In the numerical example we were considering an impact of a 60 tons ship with the velocity of 6 m/s. The final kinetic energy of the ship was compared in three cases. In the reference case the FORTRAN subroutine was not applied and the final ship velocity was close to the initial one (cf. Fig. 19a). We conclude that the amount of energy transferred to the wind turbine tower was insignificant. In the second example

\[
C_v = \begin{cases} 
1 \times 10^4 & \text{for } V < 0 \ (\text{ship approaching}) \\
2 \times 10^5 & \text{for } V > 0 
\end{cases} \quad E = \begin{cases} 
150 \text{MPa} & \text{for } V < 0 \\
1.5 \text{MPa} & \text{for } V > 0 
\end{cases} \quad (79a,b)
\]
dissipation was performed by releasing the pressure by opening valves in the chamber. It was found that the energy of the ship was reduced by 41%. In the last example considered, the dissipation was due to pressure release and change of AIS partitions stiffness. In reality it can be executed by applying pistons with controllable valves as AIS partitions. The performed change of partitions stiffness causes that the energy accumulated in a strongly tensioned front AIS wall was transferred to the kinetic energy of the ship but was changed into kinetic and strain energy of other parts of inflatable structures. By applying this method the velocity of the ship was decreased to 1.32 m/s which means a reduction by 78%. Results concerning ship velocity and system kinetic energy are presented in Fig. 19 (energy is scaled by $h^{2D} / h^{3D}$ due to ship mass reduction, cf. 74).
Figure 19. Change of ship velocity and system kinetic energy: a) no dissipation, b) dissipation by pressure release, c) dissipation by pressure release and change of partitions stiffness.
4.5.2 Mitigation of forces acting on the ship

In this section the purpose of the pressure adjustment was to achieve the lowest value of ship accelerations. The most profitable situation is stopping the ship before hitting the tower wall by using constant accelerations of minimal value. In the case of rigid piston considered in Sec. 4.2 the pressure causing such constant accelerations was also constant due to linear relation between pressure and force acting on the piston, cf. (67). In the case of ship hitting the wind turbine, pressure required to achieve constant ship accelerations is not constant due to various areas of contact between the ship and inflatable structure and forces coming from rubber deformation.

The impact considered is the same as in previous case so the mass of the ship of 60 tons is hitting the structure with the velocity of 6m/s. The initial value of pressure applied is equal to 0.21 MPa which kept on the constant level stops the ship just before approaching the tower. For such initial pressure the chamber is expanded by 0.4 m so that it is 1.1 meter wide at the moment of the ship impact. In this case maximum ship acceleration is equal to 34 m/s². To stop the ship with a constant decelerating force we have to use the acceleration of \( a^* = V_0^2 / 2S = 16.4 \text{m/s}^2 \) during the time of impact \( t^* = 2S / V = 0.34s \). The decrease of ship velocity and the ship displacements are given by the standard formulae:

\[
V^*(t) = V_0 - a^* t; \quad u^*(t) = V_0 t - 1/2 a^* t^2. \tag{80a,b}
\]

A simplified calculation of appropriate change of pressure can be performed in two manners. In the first approach, we calculate directly an additional force considering the actual state of the system (81a) and the state after modification (81b):

\[
Ma(t) - P(t) = 0 \tag{81a}
\]

\[
M[a(t) + \Delta a] - P(t) - A\Delta p = 0 \tag{81b}
\]

where \( P(t) \) are the actual forces coming from deformed rubber and actual pressure in the structure, \( \Delta p \) is the additional pressure and \( A \) is area of contact between the ship and inflatable structure. By substracting these two equations, we obtain:

\[
\Delta p(t) = \frac{M\Delta a}{A} = \frac{M(a^* - a(t))}{A} \tag{82}
\]

In the second approach, a change of pressure is based just on the difference between the optimal and actual ship acceleration, length of the time increment and scaling
coefficient \(k\). Calculation of new pressure is based on the previous pressure value according to the formula:

\[
p(t) = p_{\text{old}} + \Delta p = p_{\text{old}} + k\Delta t[a^* - a(t)]
\]  

(83)

It is necessary to take into account the length of the time step, since the automatic time incrementation is applied and lengths of the time steps vary significantly. The coefficient \(k\) has to be found basing on experience from the conducted analysis.

The most convenient method of numerical implementation is controlling pressure directly instead of changing valve coefficient. Active control of the pressure boundary condition according to the actual results of the analysis can be performed by means of the following subroutine:

0. Initialisation of the data: subroutine file ctrl_disp.for contains desired acceleration and scaling coefficient \(k\); file disp_out.dat contains initial pressure in the cavity and initial time, file urdfil_out.dat contains initial acceleration of the ship

1. First step: inflating the structure by using pressure boundary condition and assumed change of temperature (the subroutine is not applied here)

2. Second step: removing pressure boundary condition and activation of the subroutine

3. DISP subroutine reads the accelerations of the ship from the file urdfil_out.dat, old values of pressure and time from file disp_out.dat. It calculates new pressure due to Eq. (83). New values of pressure and time are written to disp_out.dat

4. The increment of the ABAQUS analysis is executed. Pressure calculated by DISP inside the chamber is used to calculate structure response (and ship acceleration)

5. When the calculations for one time increment are done subroutine URDFIL reads the actual value of ship displacements and writes it to the file urdfil_out.dat

6. The steps 3 to 5 are repeated until all increments in the step are executed
The results achieved by applying this subroutine are presented in Fig. 20. The first case concerns the initial pressure of 0.21MPa. In this case the accelerations are quite low during initial time of impact. The highest accelerations are achieved when the AIS partitions are moved to both sides and front AIS wall is strongly tensioned. The second curve is obtained by FORTRAN subroutine. Pressure increases strongly at the initial stage of impact, which helps to achieve desired level of accelerations. At the moment when deformed rubber strongly breaks the ship the pressure decreases below the initial level. It can be observed that maximum value of accelerations was decreased about 37%. To obtain the assumed level of accelerations equal to $a^* = 16.4 \text{ m/s}^2$ we had to apply very sudden changes of pressure which caused convergence problems in the analysis.

### 4.6 Alleviation of tower response

#### 4.6.1 Mitigation of local internal forces in the tower

The subsequent problem considered was minimisation of the local forces occurring in the front tower wall during collision. It is clear that these forces are reduced significantly by avoiding direct contact of the ship with the wind turbine tower. However, various changes of pressure can be applied in order to stop the ship and various maximum forces in the tower will appear. Due to the fact that inertia of the tower wall is relatively small maximal stresses will depend mainly on the maximal value of pressure. Thus, the most beneficial solution is keeping pressure on the constant level during the whole impact. This conclusion makes the problem relatively easy and not requiring the application of FORTRAN subroutine.
To find the valve coefficient we need to perform the following tasks:

- compute value of constant pressure that stops the ship after compressing whole chamber
- find change of chamber volume during impact
- calculate mass of the gas which is necessary in the chamber to obtain desired pressure and volume change
- calculate the valve coefficient that provides appropriate mass change under given conditions of pressure difference.

Numerical example was performed for the impact of the 40 tons ship with the initial velocity of 7 m/s. The pressure stopping the ship just before the tower wall was found numerically. The following pressures were obtained: 0.19 MPa in the main chamber, 0.09 MPa in the adjacent chambers and 0.04 in the other ones. The main chamber is crucial for the overall response so it will be analysed precisely. Volume of the main chamber is presented in Fig. 21a.

The deployment process was assumed to be performed during initial 200 ms of the analysis and pressure increase was linear. The change of volume obtained from numerical analysis (cf. Fig. 21a) was used to calculate mass of the gas in the chamber from ideal gas law (2):

\[
m(t) = \frac{(p(t) + p_\text{atm})V(t)}{R\bar{\theta}}
\]  \hspace{1cm} (84)

Mass of the gas is increasing strongly during the inflation stage, then it is almost equal (oscillations are due to rubber vibrations), it is reduced strongly when the ship is crushing the chamber and increasing back when the ship is bounced. The mass flow rate can be calculated as:

\[
q(t) = \frac{dm(t)}{dt} = \frac{1}{R\bar{\theta}} \frac{d(p(t)V(t))}{dt}
\]  \hspace{1cm} (85)
Figure 21. a) change of chamber volume, b) mass of the gas in the cavity, c) mass flow rate, d) inverse of flow resistance coefficient.

The function of mass flow rate was subjected to high frequency filter to eliminate excessive oscillations. Positive mass flow rate indicates flow from the internal chamber with higher pressure (0.38 MPa) to the main chamber and negative flow rate indicates flow to the environment. The last plot represents the inverse of the flow rate which is proportional to the area of the valve orifice, according to:

\[
1 / C_v = \frac{A(t)}{C_v^*} = \frac{q(t)}{\Delta p(t)}
\]  

(86)

and \(C_v^*\) indicates the valve coefficient independent on valve area.
Maximal tensile stresses arising in the tower wall adjacent to the main chamber were observed, cf. Fig. 22. In the case when inflatable structure is not applied, the stresses achieve extremely high value of 1970 MPa (assuming linear elasticity). In the case when initial pressure is adjusted such that the ship is stopped by crushing the whole chamber, but the valve remains closed maximal stresses achieve 229 MPa. Finally, when applying flow adjustment according to Fig. 21 we are able to reduce maximal stresses to 104 Mpa.

4.6.2 Alleviation of the tower vibrations

Finally our goal was to minimise the tower displacements. Alternatively it can be understood as minimisation of the energy transmitted to the tower during impact. The simplified analysis performed in Sec. 4.2 proved that for given properties of the wind turbine tower the impact time is generally too short to achieve a significant change of tower response. Anyway, a reasonable approach which can be applied for other offshore structures will be proposed. Two methods will be applied: optimisation of pressure boundary condition and optimisation of the flow coefficient.

Optimization of pressure boundary condition

The general idea is to consider several functions describing change of pressure in the chambers for which the ship is stopped after compressing maximal volume of the pneumatic structure. Then, from all these functions we have to choose the one with the smallest amplitude of tower vibrations. This main idea will be simplified by confining the class of considered functions and doing the following steps:

- We assume three functions describing the pressure boundary condition: constant, linearly decreasing and linearly increasing, cf. Fig. 23.
− We adjust the value of initial pressure amplitude and time of the impulse in a way that the ship is stopped close to the tower.

− Having FEM analysis executed, we check which function was the most beneficial for the mitigation of vibrations.

![Figure 23. Changes of pressure [Pa] applied during the analysis.](image)

The distribution of pressure over chambers was the following: middle chamber: pressure $p$; adjacent chambers: pressure $0.5 \, p$, other chambers: $0.2 \, p$. All the cases of the pressure change are presented in Fig. 23 and are described below:

0. The case when no flow from the chamber occurs (reference case)

1. After the step of inflation the pressure remains constant. The maximum displacement of the tower top is similar as in the reference case.

2. The sudden increase of pressure just before the impact (0.32 s – 0.35 s) and gradual decrease to the value $0.2 \, p_{\text{max}}$ at time 0.55 s. We obtained 16% decrease of the maximal tower displacement.

2b. The applied increase of pressure was more sudden than before. The maximum pressure was two times higher than before, end time of 0.55 s was decreased to 0.38s. The result was that the tower displacements did not decrease and stresses had grown.

3. Linear growth of pressure to 0.54 MPa during the time 0.35 s – 0.55 s. After applying the impact, excessive deformation of the front chambers occurs. Tower displacements are relatively high.
Table 6. Response of the structure for various change of pressure during impact.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Initial pressure [Pa]</th>
<th>Final pressure [Pa]</th>
<th>Displacement [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.00E+04</td>
<td>9.341E+05</td>
<td>-0.00431</td>
</tr>
<tr>
<td>1</td>
<td>2.10E+05</td>
<td>2.100E+05</td>
<td>-0.00420</td>
</tr>
<tr>
<td>2a</td>
<td>3.60E+05</td>
<td>7.200E+04</td>
<td>-0.00361</td>
</tr>
<tr>
<td>2b</td>
<td>7.20E+05</td>
<td>1.440E+05</td>
<td>-0.00390</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>5.400E+05</td>
<td>-0.00543</td>
</tr>
</tbody>
</table>

In all cases considered displacements of the tower were very similar, cf. Table 6. The maximum benefit in decreasing the tower vibrations was quite small (15%) and was achieved by a sudden rise of pressure at the initial stage of impact. The maximum displacement of the tower occurs after the impact is over and the ship is bounced from the structure. Thus, there is no point in observing the tower displacement during impact for the purpose of active control of pressure.

**Optimisation of valve properties**

This approach is based on conducting the analysis with various flow coefficients. Initial pressure in the main chamber was assumed relatively high to make the outflow of the gas possible. In each case the valve was opened only for the period when the ship is in contact with AIS and it is approaching the tower (time: 0.35–0.55 s).

Pressure impulses are presented in Fig. 24. In initial case the flow coefficient is large which indicates closed valve. Pressure impulse is similar to sine function and its amplitude is relatively high. When we decrease the flow coefficient, maximum value of pressure becomes lower and the whole pressure impulse is more flat. In the last case pressure in the chambers becomes too low and ship finally hits the wind turbine tower.

![Figure 24. Changes of pressure [Pa] applied during the analysis.](image-url)
Table 7. Response of the structure for various flow coefficients.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Cv</th>
<th>Pressure</th>
<th>Final pressure</th>
<th>Displacement [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100 E+10</td>
<td>2.10E+05</td>
<td>7.574E+05</td>
<td>-0.00431</td>
</tr>
<tr>
<td>1</td>
<td>100 E+5</td>
<td>2.10E+05</td>
<td>7.078E+05</td>
<td>-0.00422</td>
</tr>
<tr>
<td>2</td>
<td>25 E+5</td>
<td>2.10E+05</td>
<td>5.772E+05</td>
<td>-0.00397</td>
</tr>
<tr>
<td>3</td>
<td>5 E+5</td>
<td>2.10E+05</td>
<td>2.720E+05</td>
<td>-0.00365</td>
</tr>
<tr>
<td>4</td>
<td>1 E+5</td>
<td>2.10E+05</td>
<td>2.437E+05</td>
<td>-0.00378</td>
</tr>
</tbody>
</table>

The analysis performed confirms the previous results. Maximum reduction of the tower displacement was equal 15.5%, cf. Table 7. In the case of considered wind turbine tower significant reduction of tower vibrations by using inflatable structure is not feasible. Anyway, it can be a promising method for the structures with a higher period of vibrations.

4.7 Periodic collisions of the ship against the tower

Finally, we will consider periodic collisions of the ship against the wind turbine tower which occur when the ship is docked to the tower, but strong wind and waves are causing that it is pushed against the tower. In this situation subsequent impacts of the ship are not as strong as the single impact considered previously. They can be as well effectively mitigated by the inflatable structure.

In a numerical model, presented in Fig. 25, the rope connecting the ship to the wind turbine is modelled by two additional elements which have stiffness only in tension, but not in compression. The ship can approach the tower without any forces acting from the rope. When the ship is bounced from the tower the rope is tensioned, it stops the ship and it pulls the ship back to the tower.

Figure 25. Model of the ship connected to the wind turbine tower.
The procedure for adjusting pressure should be based on observing position and velocity of the ship. A general scheme of the procedure can be as follows:

1. Initial chamber deployment appropriate to stop the ship and avoid hitting tower wall. Initial pressure can be based on ship velocity measurement.

2. Change of pressure during crushing of the inflatable structure to minimise forces acting on the ship (based on ship acceleration measurement) or to alleviate stresses in the tower wall (constant pressure).

3. Pressure release at the moment of stopping the ship to dissipate impact energy accumulated in the compressed gas and avoid bouncing of the ship.

4. Consecutive deployment of the inflatable structure for the purpose of preparation to the subsequent impacts which energies may differ from initial one.
5. Conclusions and future steps

Conclusions concerning modelling of the inflatable structures in ABAQUS:

− Both ABAQUS/Standard and ABAQUS/Explicit provide convenient features and tools to model inflatable structures and release of pressure.
− ABAQUS/Explicit contains more built-in options for modelling various deployment and flow conditions.
− ABAQUS/Standard is better suited for modelling Adaptive Inflatable Structures due to the possibility of using FORTRAN subroutines for the purpose of controlling gas flow during analysis.

Conclusions concerning feasibility study of using Adaptive Inflatable Structures for protecting wind turbines and other offshore structures:

− Inflatable Structure surrounding the tower can effectively protect offshore wind turbine against ship collisions.
− By using inflatable structure we avoid direct collision between the ship and turbine tower and arising huge accelerations and forces.
− By adjusting initial pressure and controlling its release we can adapt the inflatable structure for various impact conditions and significantly increase its effectiveness.
− Controlled release of pressure helps to dissipate major part of impact energy and avoid bouncing the ship from the wind turbine.
− By controlling a flow through the valve we can precisely minimise stresses in the tower wall and forces acting on the ship.
− Inflatable structure helps to minimise vibrations of the structures of period of vibrations comparable to impact time.

Possible future steps for making the study more reliable:

− Three dimensional model of the tower surrounded by inflatable structure is desired to analyse the impact more precisely.
− The model should take into account the differences of pressure across each chamber at given instant of time.
− Description of the flow should be adjusted to the experimental data.
− More precise modelling of the composite rubber material reinforced by steel rods is desired.
− The possibilities of opening and closing the valve within the time of impact (~200 ms) should be examined.

− Finally, an experimental verification of the whole system is necessary to test out its functioning and effectiveness.
References

1) ABAQUS ver. 6.5 2005 User’s Manual. ABAQUS Inc. Rising Sun Mills, USA.


Other literature


Appendix A: MATLAB file generating 2D model of inflatable structure for ABAQUS/STANDARD (fluid-filled cavities)

format compact

clc
clear
cclc

% Parameters

r1 = 2.1; % radius of the tower
r2 = 2.8; % radius of the whole structure
n1 = 72; % number of nodes around the tower and APS
n2 = 9;  % number of APS chambers
s = 6;   % number of stiffeners

E_rubber_1 = 150.E6;    % Young modulus of rubber
E_rubber_2 = 150.E6;    % Young modulus of rubber partitions

E_sdm = 9.042e13*0.001584/50    % Tower stiffness
m2 = 4.653e10*0.001584/50       % Tower mass

alpha = 0 % Rayleigh damping for steel and rubber
beta = 0

alpha_sdm = 0.0112 % Rayleigh damping for spring-dashpot material
beta_sdm = 0.0022

theta_z = -273.15 % absolute zero temperature
p_a = 100e3   % ambient pressure – 101360

theta_r = 20 % reference temperature
p_r = 0 % reference pressure
rho_r = 1.185706 % reference density

theta_0 = 20 % initial temperature
p1_0 = 80e3 % initial pressure in front chamber (gauge pressure)
p2_0 = 50e3 % initial pressure in right and left chamber
p3_0 = 20e3 % initial pressure in other chambers
p_in = 300e3 % pressure in the accumulator
p_out = 0e3 % outside gauge pressure
Cv1 = 100e8  % flow from the central chamber
Cv2 = 100e8  % flow to the central chamber
Cv3 = 100e8  % flow from central to the upper chamber
Cv4 = 100e8  % flow from central to the lower chamber

MS = 1  % method of the solution: 1 – STD, 2 – XPL
id = 0.5  % impact distribution (jaka czesc przedniej komory jest poddana uderzeniu)
m = 80e3/50  % mass applied to the structure
V = -4  % initial velocity
dist = 5  %
length = 6

t1 = 0.2  % time of analysis
nts1 = 100  % number of time steps
t2 = 1
nts2 = 10e3

fid = fopen('c:\temp\flow_ff_con.inp','w');
fprintf(fid,'*HEADING 
');
fprintf(fid,'Two-dimensional wind turbine shielded by rubber pressurized structure 
');
fprintf(fid,'Cylindrical coordinate system 
');
fprintf(fid,'S.I. Units (N, kg, m, s, degrees Celsius) 
');
fprintf(fid,'*PREPRINT, ECHO=YES, MODEL=YES, HISTORY=YES 
');
fprintf(fid,'**
');
fprintf(fid,'*PHYSICAL CONSTANTS, ABSOLUTE ZERO=%4.2f ', theta_z);
%
fprintf(fid,'**-----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** Nodal coordinates
');
fprintf(fid,'**
');
fprintf(fid,'*NODE, SYSTEM=C, NSET=CENTER_n
');
fprintf(fid,'1   ,  0.,    0.,   0.
');
fprintf(fid,'*NODE, SYSTEM=C, NSET=SUPPORT_n
');
fprintf(fid,'2   ,  %3.3f,   180.,   0.
', 1.4*r2);
fprintf(fid,'*NODE, SYSTEM=C, NSET=SHIP_n 
');
fprintf(fid,'999  ,  %3.3f,    0.,   0. 
', dist);
fprintf(fid,'*NODE, SYSTEM=C, NSET=PIN_n 
');
fprintf(fid,'777  ,  %3.3f,    0.,   0. 
', dist);
fprintf(fid,'*NODE, SYSTEM=C, NSET=TOWER_WALL_n
');

A2
fprintf(fid,'%4d, %3.1f,   0.,  0.
', r1*1000, r1 );
fprintf(fid,'%4d, %3.1f,  %3d,  0.
', r1*1000+n1-1, r1, 360-360/n1);
fprintf(fid,'*NGEN, LINE=C, SYSTEM=C, NSET=TOWER_WALL_n
');
fprintf(fid,'%4d, %4d, %2d, 1, 0., 0., 0., 0., 0., 1\n\n', r1*1000, r1*1000+n1-1, 1 );
%
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'*NODE, SYSTEM=C, NSET=APS_WALL_n
   ');fprintf(fid,'%4d, %3.1f,   0.,  0.
', r2*1000, r2 );
fprintf(fid,'%4d, %3.1f,  %3d,  0.
', r2*1000+n1-1, r2, 360-360/n1 );
fprintf(fid,'*NGEN, LINE=C, SYSTEM=C, NSET=APS_WALL_n
');
fprintf(fid,'%4d, %4d, %2d, 1, 0., 0., 0., 0., 0., 1\n\n', r2*1000, r2*1000+n1-1, 1);
%
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'*NODE, SYSTEM=C, NSET=ADDITIONAL_n
');fprintf(fid,'%4d, %3.1f,  %3.1f,  0.
', 10000+r2*1000, r2, 360/n1/2 );
fprintf(fid,'%4d, %3.1f,  %3.1f,  0.
', 10000+r2*1000+n1/d/2-1, r2, (n1/d-1)*360/n1/2 );
fprintf(fid,'%4d, %3.1f,  %3.1f,  0.
', 10000+r2*1000+n1-n1/d/2, r2, 360-(n1/d-1)*360/n1/2 );
fprintf(fid,'%4d, %3.1f,  %3.1f,  0.
', 10000+r2*1000+n1-1, r2, 360-360/n1/2 );
fprintf(fid,'*NGEN, LINE=C, SYSTEM=C, NSET=ADDITIONAL_n
');
fprintf(fid,'%4d, %4d, %2d, 1, 0., 0., 0., 0., 0., 1\n\n', 10000+r2*1000, 10000+r2*1000+n1/d/2-1 , 1);
fprintf(fid,'%4d, %4d, %2d, 1, 0., 0., 0., 0., 0., 1\n\n', 10000+r2*1000+n1-n1/d/2, 10000+r2*1000+n1-1, 1);
%
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'*NSET, NSET=MASS_n
');fprintf(fid,'%4d
', r2*1000);
%for i=1 : 2 : id*n1/d
    fprintf(fid,'%4d \n\n', 10000+r2*1000+(i-1)/2);
% end
for i=2 : 2 : id*n1/d
    fprintf(fid,'%4d \n\n', 10000+r2*1000+i/2);
% end
%
fprintf(fid,'**
');
fprintf(fid,'*NSET, NSET=MASS1_n
');fprintf(fid,'%4d
', r2*1000);
%
fprintf(fid,'*NODE, SYSTEM=C, NSET=REFERENCE_n \n');
fprintf(fid,'%4d, %3.2f,  0,  0. \n', 201, 1.2*r2);   % wezel 201
fprintf(fid,'%4d, %3.2f, -0,  0. \n', 301, 0.6*r1);   % wezel 301
for i = 1:d
    fprintf(fid,'%4d, %3.1f, %3.1f ,  0. \n', 100+i, (r1+r2)/2, (i-1)*360/d );
end
%
fprintf(fid,'**-----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** Element connectivity\n');
fprintf(fid,'**
');
 fprintf(fid,'*ELEMENT, TYPE=B21, ELSET=TOWER_WALL_e\n');
 fprintf(fid,'%4d, %4d, %4d
',2001, r1*1000, r1*1000+1);           % tower wall el 2001
 fprintf(fid,'%4d, %4d, %4d
',2000+n1, r1*1000+n1-1, r1*1000);        % tower wall el 2000
 fprintf(fid,'*ELGEN, ELSET=TOWER_WALL_e\n');
 fprintf(fid,'%4d, %4d, %4d, 1
',2001, n1-1, 1);
%
fprintf(fid,'**
');
fprintf(fid,'**
');
 fprintf(fid,'*ELEMENT, TYPE=B21, ELSET=APS_WALL_e\n');
 fprintf(fid,'%4d, %4d, %4d
',4001+n1/d/2, r2*1000+n1/d/2, r2*1000+n1/d/2+1);  % APS WALL el 4001
 fprintf(fid,'*ELGEN, ELSET=APS_WALL_e\n');
 fprintf(fid,'%4d, %4d, %4d, 1
',4001+n1/d/2, n1-n1/d, 1);
%
fprintf(fid,'**
');
fprintf(fid,'**
');
 fprintf(fid,'*ELEMENT, TYPE=B21, ELSET=APS_WALL_e\n');
 fprintf(fid,'%4d, %4d, %4d
',4401, r2*1000+n1-n1/d/2, 10000+r2*1000+n1-n1/d/2+1);  % APS WALL el 4401
 fprintf(fid,'%4d, %4d, %4d
',4402, 10000+r2*1000+n1-n1/d/2, r2*1000+n1-n1/d/2+1);  % APS WALL el 4402
 fprintf(fid,'%4d, %4d, %4d
',4400+n1/d+1, r2*1000, 10000+r2*1000 ); % APS WALL el 4400
 fprintf(fid,'%4d, %4d, %4d
',4400+n1/d+2, 10000+r2*1000, r2*1000+1 ); % APS WALL el 4400
 fprintf(fid,'%4d, %4d, %4d
',4400+n1/d, 10000+r2*1000+n1-1, r2*1000);  % APS WALL el 4400
 fprintf(fid,'*ELGEN, ELSET=APS_WALL_e\n');
 fprintf(fid,'%4d, %4d, %4d, %4d 
',4401, n1/d/2, 1, 2);  % APS WALL el 4401
 fprintf(fid,'%4d, %4d, %4d, %4d 
',4402, n1/d/2-1, 1, 2); % APS WALL el 4402
 fprintf(fid,'%4d, %4d, %4d, %4d 
',4400+n1/d+1, n1/d/2, 1, 2); % APS WALL el 4400
 fprintf(fid,'%4d, %4d, %4d, %4d \n',4400+n1/d+2, n1/d/2, 1, 2); % APS WALL el 4400
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'*ELEMENT, TYPE=B21, ELSET=PARTITIONS_e
');
fprintf(fid,'%4d, %4d, %4d
',3001, r1*1000+n1/d/2, r2*1000+n1/d/2);
fprintf(fid,'*ELGEN, ELSET=PARTITIONS_e
');
fprintf(fid,'%4d, %4d, %4d, 1
',3001, d, n1/d);
%
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'*ELEMENT, TYPE=B21, ELSET=STIFFENERS_e
');
for i = 1:s
    fprintf(fid,'%4d, %2d, %4d
', 1000+i, 1, r1*1000+n1/s/2+(i-1)*n1/s);
end
%
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'*ELEMENT, TYPE=B21, ELSET=SPRING_DASHPOT_e
');
fprintf(fid,'1, 1, 2
');
%
fprintf(fid,'*ELEMENT, TYPE=mass, ELSET=MASS2_e
');
fprintf(fid,'33, 1 
');
%
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'*ELEMENT, TYPE=mass, ELSET=MASS_e
');
for i=1 : 2 : id*n1/d
    fprintf(fid,'%4d, %4d 
', 4*i, 10000+r2*1000+(i-1)*1/2);
end
for i=2 : 2 : id*n1/d
    fprintf(fid,'%4d, %4d 
', 4*i+2, r2*1000+i/2);
end
%
fprintf(fid,'*ELSET, ELSET=HST_e 
');
fprintf(fid,'%4d, %4d 
', 2001, 2000+n1);
fprintf(fid,'*ELSET, ELSET=HSR_e 
');
fprintf(fid,'%4d, %4d 
',4400+n1/d, 4400+n1/d+1  );
%
fprintf(fid,'**-----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** Fluid elements\n');
fprintf(fid,'**\n');

% fprintf(fid,'*ELEMENT, TYPE=F2D2, ELSET=FLUID_e\n');
fprintf(fid,'%4d, %4d, %4d\n',5001, r1*1000+1, r1*1000);
fprintf(fid,'%4d, %4d, %4d\n',5000+n1, r1*1000, r1*1000+n1-1);
fprintf(fid,'*ELGEN, ELSET=FLUID_e\n');
fprintf(fid,'%4d, %4d, %4d\n',5001, n1-1, 1);
% fprintf(fid,'**\n');
fprintf(fid,'**\n');

% fprintf(fid,'*ELEMENT, TYPE=F2D2, ELSET=FLUID_e\n');
fprintf(fid,'%4d, %4d, %4d\n',6001, r1*1000+n1/d/2, r2*1000+n1/d/2);
fprintf(fid,'%4d, %4d, %4d\n',6001, d, n1/d);
fprintf(fid,'*ELGEN, ELSET=FLUID_e\n');
fprintf(fid,'%4d, %4d, %4d\n',7001+n1/d/2, r2*1000+n1/d/2, r2*1000+n1/d/2+1);
fprintf(fid,'%4d, %4d, %4d\n',7001+n1/d/2, n1-n1/d, 1);
fprintf(fid,'%4d, %4d, %4d\n',7401, r2*1000+n1-n1/d/2, 10000+r2*1000+n1-n1/d/2);
fprintf(fid,'%4d, %4d, %4d\n',7402, 10000+r2*1000+n1-n1/d/2, r2*1000+n1-n1/d/2+1);
fprintf(fid,'%4d, %4d, %4d\n',7400+n1/d+1, r2*1000, 10000+r2*1000);
fprintf(fid,'%4d, %4d, %4d\n',7400+n1/d+2, n1/d/2, 1);
fprintf(fid,'%4d, %4d, %4d\n',7400+n1/d, 10000+r2*1000+n1-1, r2*1000);
fprintf(fid,'*ELGEN, ELSET=FLUID_e\n');
fprintf(fid,'%4d, %4d, %4d\n',7401, n1/d/2, 1, 2);
fprintf(fid,'%4d, %4d, %4d\n',7402, n1/d/2-1, 1, 2);
fprintf(fid,'%4d, %4d, %4d\n',7400+n1/d+1, n1/d/2, 1, 2);
fprintf(fid,'%4d, %4d, %4d\n',7400+n1/d+2, n1/d/2, 1, 2);
fprintf(fid,'**
');
%
fprintf(fid,'*ELSET, ELSET=CAVITY1_e, GENERATE 
');
fprintf(fid,'%4d, %4d, %4d 
', 5000+n1-n1/d/2+1, 5000+n1, 1);
fprintf(fid,'%4d, %4d, %4d 
', 5001, 5000+n1/d/2, 1);
fprintf(fid,'%4d, %4d, %4d 
', 6000+d, 6000+d, 1);
fprintf(fid,'%4d, %4d, %4d 
', 7401, 7400+2*n1/d, 1);
fprintf(fid,'%4d, %4d, %4d 
', 8001, 8001, 1);
%
for i=2:d
    fprintf(fid,'*ELSET, ELSET=CAVITY%d_e, GENERATE 
', i );
    fprintf(fid,'%4d, %4d, %4d 
', 5001+n1/d/2+(i-2)*n1/d, 5000+3*n1/d/2+(i-2)*n1/d, 1);
    fprintf(fid,'%4d, %4d, %4d 
', 6000+i-1, 6000+i-1, 1 );
    fprintf(fid,'%4d, %4d, %4d 
', 7001+n1/d/2+(i-2)*n1/d, 7000+3*n1/d/2+(i-2)*n1/d, 1);
    fprintf(fid,'%4d, %4d, %4d 
', 8000+i, 8000+i, 1);
end
%
fprintf(fid,'*ELEMENT, TYPE=FLINK, ELSET=FFLOW1_e 
');
fprintf(fid,'9001, 101, 201 
');
fprintf(fid,'*ELEMENT, TYPE=FLINK, ELSET=FFLOW2_e 
');
fprintf(fid,'9002, 101, 301 
');
fprintf(fid,'*ELEMENT, TYPE=FLINK, ELSET=FFLOW3_e 
');
fprintf(fid,'9003, 101, 102 
');
fprintf(fid,'*ELEMENT, TYPE=FLINK, ELSET=FFLOW4_e 
');
fprintf(fid,'9004, 101, %4d 
', 100+d);
%
fprintf(fid,'*ELSET, ELSET=FFLOW_e 
' );
fprintf(fid,'9001 
');
fprintf(fid,'9001, 9002, 9003, 9004 
');
%
fprintf(fid,'** Element properties
');
fprintf(fid,'**

*BEAM SECTION, ELSET=TOWER_WALL_e, SECTION=RECT, MATERIAL=STEEL 
');
fprintf(fid,'0.05, 0.05 
');
fprintf(fid,'*BEAM SECTION, ELSET=APS_WALL_e , SECTION=RECT, MATERIAL=RUBBER_1 
');
fprintf(fid,'0.04, 0.01 
');
fprintf(fid,'*BEAM SECTION, ELSET=PARTITIONS_e, SECTION=RECT, MATERIAL=RUBBER_2
');
fprintf(fid,'0.04, 0.04 
');
fprintf(fid,'*BEAM SECTION, ELSET=STIFFENERS_e, SECTION=RECT, MATERIAL=STEEL 
');
fprintf(fid,'0.1, 0.1 
');
fprintf(fid,'*BEAM SECTION, ELSET=SPRING_DASHPOT_e, SECTION=RECT, MATERIAL=SDM 
');
fprintf(fid,'0.1, 0.1 
');
fprintf(fid,'**
');
fprintf(fid,'**
');
%
if MS==1
fprintf(fid,'*MASS, ELSET=MASS2_e, ALPHA= %4.4f 
', alpha_sdm);
fprintf(fid,'%4d 
', m2);
else
fprintf(fid,'*MASS, ELSET=MASS2_e 
');
fprintf(fid,'%4d 
', m2);
end
fprintf(fid,'*MASS, ELSET=MASS_e 
');
fprintf(fid,'%4.6f 
', m/(2*id*n1/d+1));
fprintf(fid,'**
');
fprintf(fid,'**
');
%
for i=1:d
fprintf(fid,'*FLUID PROPERTY, ELSET=CAVITY%d_E, REF NODE=%d, TYPE=PNEUMATIC,
AMBIENT=%4.2f 
', i, 100+i, p_a);    
fprintf(fid,'0.04 
');
fprintf(fid,'*FLUID DENSITY, PRESSURE=%4.2f, TEMPERATURE=%4.2f 
', p_r, theta_r);
fprintf(fid,'%4.2f 
', rho_r);
end
%
fprintf(fid,'*FLUID LINK, ELSET=FFLOW1_e, TYPE=FUNCTION 
');
fprintf(fid,'%4.2f 
', Cv1);
fprintf(fid,'*FLUID LINK, ELSET=FFLOW2_e, TYPE=FUNCTION 
');
fprintf(fid,'%4.2f 
', Cv2);
fprintf(fid,'*FLUID LINK, ELSET=FFLOW3_e, TYPE=FUNCTION 
');
fprintf(fid,'%4.2f 
', Cv3);
fprintf(fid,'*FLUID LINK, ELSET=FFLOW4_e, TYPE=FUNCTION 
');
fprintf(fid,'%4.2f 
', Cv4);
%
fprintf(fid,'**-----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** Materials 
');
fprintf(fid,'**
');
fprintf(fid,'*MATERIAL, NAME=STEEL 
');
fprintf(fid,'*Density 
');
fprintf(fid,'7850. 
');
fprintf(fid,'*Elastic 
');
fprintf(fid,'200.E9, 0.3 
');
fprintf(fid,'*DAMPING, ALPHA= %3.2f, BETA= %3.2f \n', alpha, beta);
%
fprintf(fid,'*MATERIAL, NAME=RUBBER_1 
');
fprintf(fid,'*Density 
');
fprintf(fid,'2000. 
');
fprintf(fid,'*Elastic 
');
fprintf(fid,'%4g, 0.49932 \n', E_rubber_1);
fprintf(fid,'*DAMPING, ALPHA= %3.2g, BETA= %3.2g \n', alpha, beta);
%
fprintf(fid,'*MATERIAL, NAME=RUBBER_2 
');
fprintf(fid,'*Density 
');
fprintf(fid,'2000. 
');
fprintf(fid,'*Elastic 
');
fprintf(fid,'%4g, 0.49932 \n', E_rubber_2);
fprintf(fid,'*DAMPING, ALPHA= %3.2g, BETA= %3.2g \n', alpha, beta);
%
fprintf(fid,'*MATERIAL, NAME=SDM 
');
fprintf(fid,'*Density 
');
fprintf(fid,'2000. 
');
fprintf(fid,'*Elastic 
');
fprintf(fid,'%4g, 0.3 \n', E_sdm);
fprintf(fid,'*DAMPING, ALPHA= %3.2g, BETA= %3.2g \n', alpha_sdm, beta_sdm);
%
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'** Contact 
');
fprintf(fid,'*SURFACE, TYPE=SEGMENTS, NAME=SHIP_ms 
');
fprintf(fid,'START,  %3.3f, %3.3f   
', dist, -length/2 );

A9
fprintf(fid,'LINE , %3.3f, %3.3f   
',dist, length/2 );

fprintf(fid,'*RIGID BODY, ANALYTICAL SURFACE=SHIP_ms, PIN NSET=PIN_n, REF NODE=999
');
fprintf(fid,'*ELEMENT, TYPE=mass, ELSET=MASS_e
');
fprintf(fid,'3, 777 \n');
fprintf(fid,'*MASS, ELSET=MASS_e \n');

fprintf(fid,'*SURFACE, TYPE=ELEMENTS, NAME=APS_out_ss 
');
fprintf(fid,'APS_WALL_e, SNEG \n');
fprintf(fid,'*SURFACE, TYPE=ELEMENTS, NAME=APS_in_ss 
');
fprintf(fid,'APS_WALL_e, SPOS \n');
fprintf(fid,'*SURFACE, TYPE=ELEMENTS, NAME=TOWER_ms \n');
fprintf(fid,'TOWER_WALL_e, SNEG \n');

if MS==1
  fprintf(fid,'*CONTACT PAIR, INTERACTION=INT_1 \n');
  fprintf(fid,'APS_out_ss, SHIP_ms \n');
  fprintf(fid,'*CONTACT PAIR, INTERACTION=INT_2 \n');
  fprintf(fid,'APS_in_ss, TOWER_ms \n');
  fprintf(fid,'*SURFACE INTERACTION, NAME=INT_1 \n');
  fprintf(fid,'1.0  \n');
  fprintf(fid,'*SURFACE INTERACTION, NAME=INT_2 \n');
  fprintf(fid,'1.0  \n');
end

if MS==1
  fprintf(fid,'*CONTACT PAIR, INTERACTION=INT_1 \n');
  fprintf(fid,'APS_out_ss, SHIP_ms \n');
  fprintf(fid,'*CONTACT PAIR, INTERACTION=INT_2 \n');
  fprintf(fid,'APS_in_ss, TOWER_ms \n');
  fprintf(fid,'*SURFACE INTERACTION, NAME=INT_1 \n');
  fprintf(fid,'1.0  \n');
  fprintf(fid,'*SURFACE INTERACTION, NAME=INT_2 \n');
  fprintf(fid,'1.0  \n');
end

for i=1:d
  fprintf(fid,'%4d, %4.2f \n', 100+i, theta_0);
end

% fprintf(fid,'**-----------------------------------------------------------------------------
');
% fprintf(fid,'**
');
% fprintf(fid,'** Initial conditions \n');
% fprintf(fid,'**
');
% fprintf(fid,'*INITIAL CONDITIONS, TYPE=VELOCITY \n');
% fprintf(fid,'MASS_n, 1, %4.2g \n', V);
% fprintf(fid,'*INITIAL CONDITIONS, TYPE=TEMPERATURE \n');
% for i=1:d
%   fprintf(fid,'%4d, %4.2f \n', 100+i, theta_0);
% end
fprintf(fid, 'end\n');
fprintf(fid, 'fprintf(fid,' '%4d, %4.2f  \n', 201, theta_0);
fprintf(fid,' fprintf(fid,' '%4d, %4.2f  \n', 301, theta_0);
%
fprintf(fid,*INITIAL CONDITIONS, TYPE=FLUID PRESSURE
');
fprintf(fid,' fprintf(fid,' '101, %4.0f  \n', p1_0);
fprintf(fid,' fprintf(fid,' '201, %6.0f  \n', p_out);
fprintf(fid,' fprintf(fid,' '301, %6.0f  \n', p_in);
%for i=2:d
% fprintf(fid,' fprintf(fid,' '%d, %6.0f  \n', 100+i, p2_0);
%end
% fprintf(fid,*INITIAL CONDITIONS, TYPE=VELOCITY
');
fprintf(fid,' fprintf(fid,' '999, 1, %4d  \n', V);
%
fprintf(fid,*AMPLITUDE, NAME=PRESS_CHANGE, DEFINITION=TABULAR, SMOOTH=0
');
fprintf(fid,' fprintf(fid,' '0,0, %4.2g ,1  \n', t1);
%
fprintf(fid,**-----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** History data
');
fprintf(fid,'**
');
if MS==1
fprintf(fid,*STEP, NLGEOM=YES, INC=5000
');
fprintf(fid,'fprintf(fid,' 'Direct implicit integration
');
fprintf(fid,*DYNAMIC, ALPHA=-0.05, DIRECT
');
fprintf(fid,'fprintf(fid,' '%6f, %6f, %6f, %6f
', t1/nts1, t1, t1/10000, t1/10);
else
fprintf(fid,*STEP
');
fprintf(fid,'fprintf(fid,' 'Explicit dynamics
');
fprintf(fid,*DYNAMIC, EXPLICIT
');
fprintf(fid,'fprintf(fid,' '%6f\n', t1);
end
%
if MS==2
fprintf(fid,*CONTACT PAIR, INTERACTION=INT_1
');
fprintf(fid,'fprintf(fid,' 'APS_out_ss, SHIP_ms
');
fprintf(fid,*CONTACT PAIR, INTERACTION=INT_2
');
fprintf(fid,'fprintf(fid,' 'APS_in_ss, TOWER_ms
');
fprintf(fid,'fprintf(fid,' '%6f\n', t1);
fprintf(fid,"*SURFACE INTERACTION, NAME=INT_1 \n");
fprintf(fid,'1.0 \n');
fprintf(fid,"*SURFACE INTERACTION, NAME=INT_2 \n");
fprintf(fid,'1.0 \n');
end
%
fprintf(fid,"*BOUNDARY \n");
fprintf(fid,'2, ENCASTRE\n');
fprintf(fid,'999, 4, 6 \n');
%
fprintf(fid,"*BOUNDARY, AMPLITUDE=PRESS_CHANGE \n");
fprintf(fid,'101, 8, 8, %4.2g \n', p1_0);   % BC in the front cavity
fprintf(fid,'102, 8, 8, %4.2g \n', p2_0);   % BC in upper and lower cavity
fprintf(fid,'%4d, 8, 8, %4.2g \n', 100+d, p2_0);
for i=3:d-1
    fprintf(fid,"%d, 8, 8, %6.0f \n", 100+i, p3_0);
end
fprintf(fid,"*BOUNDARY \n");
fprintf(fid,'201, 8, 8, %4.2g \n', p_out);    % BC at reference node
fprintf(fid,'301, 8, 8, %4.2g \n', p_in);     % BC at reference node
%
fprintf(fid,"**-----------------------------------------------------------------------------
");
fprintf(fid,"**
");
fprintf(fid,"** Output
");
fprintf(fid,"**
");
fprintf(fid,"***-----------------------------------------------\n");
fprintf(fid,"***\n");
fprintf(fid,"*** Output\n");
fprintf(fid,"***\n");
%
if MS==1
    fprintf(fid,"**OUTPUT,FIELD, FREQUENCY=2 \n");
else
    fprintf(fid,"**OUTPUT,FIELD, TIME INTERVAL=0.01 \n");
end
fprintf(fid,"*NODE OUTPUT \n");
fprintf(fid,'U \n');
fprintf(fid,"*NODE OUTPUT, NSET=REFERENCE_n \n");
fprintf(fid,'PCAV \n');
fprintf(fid,'CVOL \n');
fprintf(fid,'NT \n');
fprintf(fid,"*ELEMENT OUTPUT \n");
fprintf(fid,'1, 3, 5 \n');
fprintf(fid,'S, SF \n');
%
fprintf(fid,'*OUTPUT, HISTORY, FREQUENCY=2 \n');
fprintf(fid,'*NODE OUTPUT, NSET=CENTER_n \n');
fprintf(fid,'U,a  \n');
fprintf(fid,'*NODE OUTPUT, NSET=MASS1_n \n');
fprintf(fid,'U,a  \n');
fprintf(fid,'*NODE OUTPUT, NSET=SHIP_n \n');
fprintf(fid,'U,a  \n');
fprintf(fid,'*NODE OUTPUT, NSET=SUPPORT_n \n');
fprintf(fid,'RF, \n');
fprintf(fid,'*NODE OUTPUT, NSET=REFERENCE_n \n');
fprintf(fid,'PCAV \n');
fprintf(fid,'CVOL \n');
fprintf(fid,'NT \n');
fprintf(fid,'*ELEMENT OUTPUT, ELSET=FFLOW_e \n');
fprintf(fid,'MFL \n');
fprintf(fid,'MFLT \n');
%
fprintf(fid,'*ELEMENT OUTPUT, ELSET=FLUID_e \n');
fprintf(fid,'TEMP \n');
fprintf(fid,'*ENERGY OUTPUT \n');
fprintf(fid,'ALLKE, ALLSE, ALLVD, ALLWK, ETOTAL \n');
%
fprintf(fid,'*ELEMENT OUTPUT, ELSET=HST_e \n');
fprintf(fid,'S11 \n');
fprintf(fid,'*ELEMENT OUTPUT, ELSET=HSR_e \n');
fprintf(fid,'S11 \n');
%
fprintf(fid,'**
');
fprintf(fid,'*NODE PRINT
');
fprintf(fid,'PCAV, 
');
fprintf(fid,'CVOL, 
');
fprintf(fid,'*EL PRINT
');
fprintf(fid,'S, SF \n');
%
fprintf(fid,'*MONITOR, DOF=1, NODE=CENTER_n \n');
fprintf(fid,'*END STEP\n');
%
fprintf(fid,'*-----------------------------------------------------------------------------
');
fprintf(fid,'*-----------------------------------------------------------------------------
');
%---------------------------------
% NEW STEP FOR REMOVING PRESS BC

% NEW STEP FOR REMOVING PRESS BC
if MS==1
    fprintf(fid,'*STEP, NLGEOM=YES, INC=5000 
');
    fprintf(fid,'Direct implicit integration 
');
% fprintf(fid,'*DYNAMIC, ALPHA=-0.05, DIRECT 
');
    fprintf(fid,'*DYNAMIC, ALPHA=-0.05, HAFTOL=10e4 
');
    fprintf(fid,'%4f, %4f, %4f, %4f 
', t2/nts2, t2, t2/100e8, t2/200);
else
    fprintf(fid,'*STEP 
');
    fprintf(fid,'Explicit dynamics 
');
    fprintf(fid,'*DYNAMIC, EXPLICIT 
');
    fprintf(fid,',%6f 
', t2 );
end
% if MS==1
    fprintf(fid,'*INITIAL CONDITIONS, TYPE=FLUID PRESSURE 
');
    fprintf(fid,'101, %4.0f  
', p1_0);
    fprintf(fid,'102, %4.0f  
', p2_0);
    fprintf(fid,'%4d, 8, 8, %4.2g  
', 100+d, p2_0);
    for i=3:d-1
        fprintf(fid,'%d, %6.0f  
', 100+i, p3_0);
    end
end
% fprintf(fid,'*BOUNDARY, OP=NEW 
');
fprintf(fid,'2, ENCASTRE
');
fprintf(fid,'%d, 2, 2 
', 1000*r1+n1/2);
% fprintf(fid,'*BOUNDARY   
');
fprintf(fid,'101, 8, 8, %4.2g  
', p1_0);     % BC in the front cavity
    for i=2:d
        fprintf(fid,'%d, 8, 8, %6.0f  
', 100+i, p2_0);
    end
% fprintf(fid,'*BOUNDARY  
');
fprintf(fid,'201, 8, 8, %4.2g  
', p_out);    % BC at reference node
fprintf(fid,'301, 8, 8, %4.2g  
', p_in);     % BC at reference node
% fprintf(fid,'102, 8, 8, %4.2g  
', p2_0);    % BC at reference node
% fprintf(fid,'106, 8, 8, %4.2g  
', p2_0);    % BC at reference node

%---------------------------------
%  
%
% fprintf(fid,'**-----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** Output
');
fprintf(fid,'**
');
%
if MS==1
    fprintf(fid,'*OUTPUT,FIELD, FREQUENCY=2  
');
else
    fprintf(fid,'*OUTPUT,FIELD, TIME INTERVAL=0.01  
');
end
fprintf(fid,'*NODE OUTPUT 
');
fprintf(fid,'U 
');
fprintf(fid,'*NODE OUTPUT, NSET=REFERENCE_n 
');
fprintf(fid,'PCAV 
');
fprintf(fid,'CVOL 
');
fprintf(fid,'NT 
');
fprintf(fid,'*ELEMENT OUTPUT 
');
fprintf(fid,'1, 3, 5 
');
%
fprintf(fid,'*OUTPUT, HISTORY, FREQUENCY=2 
');
fprintf(fid,'*NODE OUTPUT, NSET=CENTER_n 
');
fprintf(fid,'U,a  
');
fprintf(fid,'*NODE OUTPUT, NSET=MASS1_n 
');
fprintf(fid,'U,a  
');
fprintf(fid,'*NODE OUTPUT, NSET=SHIP_n 
');
fprintf(fid,'U,a  
');
fprintf(fid,'*NODE OUTPUT, NSET=SUPPORT_n 
');
fprintf(fid,'RF,  
');
fprintf(fid,'*NODE OUTPUT, NSET=REFERENCE_n 
');
fprintf(fid,'PCAV 
');
fprintf(fid,'CVOL 
');
fprintf(fid,'NT 
');
fprintf(fid,'*ELEMENT OUTPUT, ELSET=FFLOW_e 
');
fprintf(fid,'MFL 
');
fprintf(fid,'MFLT 
');
%
fprintf(fid,'*ELEMENT OUTPUT, ELSET=FLUID_e 
');
fprintf(fid,'TEMP 
');
fprintf(fid,'*ENERGY OUTPUT 
');
fprintf(fid,'ALLKE, ALLSE, ALLVD, ALLWK, ETOTAL \n');
%
fprintf(fid,*ELEMENT OUTPUT, ELSET=HST_e \n');
fprintf(fid,'S11 \n');
fprintf(fid,*ELEMENT OUTPUT, ELSET=HSR_e \n');
fprintf(fid,'S11 \n');
%
fprintf(fid,**\n');
fprintf(fid,*NODE PRINT\n');
fprintf(fid,'PCAV, \n');
fprintf(fid,'CVOL, \n');
fprintf(fid,*EL PRINT\n');
fprintf(fid,'S, SF \n');
%
fprintf(fid,*MONITOR, DOF=1, NODE=CENTER_n \n');
fclose(fid);
Appendix B: MATLAB file generating 2D model of inflatable structure for ABAQUS/EXPLICIT (surface-based cavities)

format compact
clc
clear

c

r1 = 2.1;     % radius of the tower
r2 = 2.8;     % radius of the whole structure
n1 = 288;      % number of nodes around the tower and APS
d = 9;        % number of APS chambers
s = 6;        % number of stiffeners

E_rubber_1 = 150.E6;      % Young modulus of rubber
E_rubber_2 = 150.E6;      % Young modulus of rubber partitions

E_sdm = 9.042e13*0.001584/50    % Tower stiffness
m2 = 4.653e10*0.001584/50       % Tower mass

alpha = 0             % Rayleigh damping for steel and rubber
beta = 0
alpha_sdm = 0.0112    % Rayleigh damping for spring-dashpot material
beta_sdm = 0.0022

theta_z = -273.15     % absolute zero temperature
p_a = 100e3           % ambient pressure – 101360

theta_0 = 20          % initial temperature
p1_0 = 80e3           % initial pressure in front chamber (gauge pressure)
p2_0 = 50e3           % initial pressure in right and left chamber
p3_0 = 5e3           % initial pressure in other chambers
p_in = 300e3          % pressure in the accumulator
p_out = 0e3           % outside gauge pressure

Cv1 = 100e8          % flow from the central chamber
Cv2 = 100e8          % flow to the central chamber (does not work)
Cv3 = 100e8          % flow from central to the upper chamber
Cv4 = 100e8          % flow from central to the lower chamber
m = \frac{40e3}{50} \quad \% \text{mass of the ship}

V = -8 \quad \% \text{initial velocity}

dist = 5 \quad \% \text{distance from the ship to the tower}

\text{length} = 6 \quad \%

ad = 0 \quad \% \text{0 – izotermic, 1 – adiabatic}

t_1 = 0.2 \quad \% \text{time of inflation}

t_2 = 1 \quad \% \text{time of the analysis}

fid = fopen('c:\temp\flow_surf_con.inp','w');

fprintf(fid,'*HEADING \n');
fprintf(fid,'Two-dimensional wind turbine shielded by rubber pressurized structure \n');
fprintf(fid,'Cylindrical coordinate system \n');
fprintf(fid,'S.I. Units (N, kg, m, s, degrees Celsius) \n');
fprintf(fid,'*PREPRINT, ECHO=YES, MODEL=YES, HISTORY=YES \n');
fprintf(fid,'**
');
fprintf(fid,'*PHYSICAL CONSTANTS, ABSOLUTE ZERO=%4.2f, UNIVERSAL GAS CONSTANT=8.314 \n', theta_z);
%    fprintf(fid,'**-----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** Nodal coordinates
');
fprintf(fid,'**
');
fprintf(fid,'*NODE, SYSTEM=C, NSET=CENTER_n
');
fprintf(fid,'1   ,  0.,    0.,   0.
');
fprintf(fid,'*NODE, SYSTEM=C, NSET=SUPPORT_n
');
fprintf(fid,'2   ,  %3.3f,   180.,   0.
', 1.4*r2);
fprintf(fid,'*NODE, SYSTEM=C, NSET=SHIP_n 
');
fprintf(fid,'999  ,  %3.3f,    0.,   0. 
', dist);
fprintf(fid,'*NODE, SYSTEM=C, NSET=PIN_n 
');
fprintf(fid,'777  ,  %3.3f,    0.,   0. 
', dist);
fprintf(fid,'*NODE, SYSTEM=C, NSET=TOWER_WALL_n
');
fprintf(fid,'%4d, %3.1f,   0.,  0.
', r1*1000, r1);
fprintf(fid,'%4d, %3.1f,  %3d,  0.
', r1*1000+n1-1, r1, 360-360/n1);
fprintf(fid,'*NGEN, LINE=C, SYSTEM=C, NSET=TOWER_WALL_n
');
fprintf(fid,'%4d, %4d, %2d, 1, 0., 0., 0., 0., 0., 1.
', r1*1000, r1*1000+n1-1, 1 );
%
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'*NODE, SYSTEM=C, NSET=APS_WALL_n
   ');fprintf(fid,'%4d, %3.1f,   0.,  0.
', r2*1000, r2 );
fprintf(fid,'%4d, %3.1f,  %3d,  0.
', r2*1000+n1-1, r2, 360-360/n1 );
fprintf(fid,'*NGEN, LINE=C, SYSTEM=C, NSET=APS_WALL_n
');fprintf(fid,'%4d, %4d, %2d, 1, 0., 0., 0., 0., 0., 1.'n', r2*1000, r2*1000+n1-1, 1);
%
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'*NODE, SYSTEM=C, NSET=ADDITIONAL_n
');fprintf(fid,'%4d, %3.1f,  %3.1f,  0.
', 10000+r2*1000, r2, 360/n1/2 );
fprintf(fid,'%4d, %3.1f,  %3.1f,  0.
', 10000+r2*1000+n1/d/2-1, r2, (n1/d-1)*360/n1/2 );
fprintf(fid,'%4d, %3.1f,  %3.1f,  0.
', 10000+r2*1000+n1-n1/d/2, r2, 360-(n1/d-1)*360/n1/2 );
fprintf(fid,'%4d, %3.1f,  %3.1f,  0.
', 10000+r2*1000+n1-1, r2, 360-360/n1/2 );
%
fprintf(fid,'*NGEN, LINE=C, SYSTEM=C, NSET=ADDITIONAL_n
');fprintf(fid,'%4d, %4d, %2d, 1, 0., 0., 0., 0., 0., 1.'n', 10000+r2*1000, 10000+r2*1000+n1/d/2-1 , 1);
fprintf(fid,'%4d, %4d, %2d, 1, 0., 0., 0., 0., 0., 1.'n', 10000+r2*1000+n1-n1/d/2, 10000+r2*1000+n1-1, 1);
%
fprintf(fid,'**
');
fprintf(fid,'*NSET, NSET=MASS1_n
');fprintf(fid,'%4d
', r2*1000);
%
fprintf(fid,'*NODE, SYSTEM=C, NSET=REFERENCE_n 
');fprintf(fid,'%4d, %3.1f,  0.,   0. 
', 201, 1.2*(r1+r2));  % wezel 201
fprintf(fid,'%4d, %3.1f,  0.,   0. 
', 301, 0.5*r1);  % wezel 301
for i = 1:d
    fprintf(fid,'%4d, %3.1f, %3.1f ,  0. 
', 100+i, (r1+r2)/2, (i-1)*360/d );
end
%
%
fprintf(fid,'**-----------------------------------------------------------------------------
');fprintf(fid,'**
');
fprintf(fid,'** Element connectivity
');fprintf(fid,'**
');fprintf(fid,'*ELEMENT, TYPE=B21, ELSET=TOWER_WALL_e
');fprintf(fid,'%4d, %4d, %4d
',2001, r1*1000, r1*1000+1);
fprintf(fid,'%4d, %4d, %4d
',2000+n1, r1*1000+n1-1, r1*1000);
fprintf(fid,*ELGEN, ELSET=TOWER_WALL_e\n*); fprintf(fid,%4d, %4d, %4d, 1\n',2001, n1-1, 1);
% fprintf(fid,**\n*); fprintf(fid,**\n*); fprintf(fid,*ELEMENT, TYPE=B21, ELSET=APS_WALL_e\n*); fprintf(fid,%4d, %4d, %4d\n*'); fprintf(fid,*ELGEN, ELSET=APS_WALL_e\n*); fprintf(fid,%4d, %4d, %4d, 1
*'); fprintf(fid,**\n*'); fprintf(fid,**\n*); fprintf(fid,*ELEMENT, TYPE=B21, ELSET=APS_WALL_e\n*); fprintf(fid,%4d, %4d, %4d
*'); fprintf(fid,*ELGEN, ELSET=APS_WALL_e\n*); fprintf(fid,%4d, %4d, %4d
*'); fprintf(fid,**\n*'); fprintf(fid,**\n*); fprintf(fid,*ELEMENT, TYPE=B21, ELSET=PARTITIONS_e\n*); fprintf(fid,%4d, %4d, %4d
*'); fprintf(fid,**\n*'); fprintf(fid,**\n*); fprintf(fid,*ELEMENT, TYPE=B21, ELSET=STIFFENERS_e\n*); for i = 1:s
fprintf(fid,%4d, %2d, %4d
*'); fprintf(fid,**\n*'); fprintf(fid,**\n*); fprintf(fid,*ELEMENT, TYPE=B21, ELSET=SPRING_DASHPOT_e\n*);
fprintf(fid,'1, 1, 2\n');
%
fprintf(fid,'*ELEMENT, TYPE=mass, ELSET=MASS2_e\n');
fprintf(fid,'33, 1 \n');
%
fprintf(fid,'*ELSET, ELSET=HST_e \n');
fprintf(fid,'%4d, %4d \n', 2001, 2000+n1 );
fprintf(fid,'*ELSET, ELSET=HSR_e \n');
fprintf(fid,'%4d, %4d \n',4400+n1/d, 4400+n1/d+1  );
%
fprintf(fid,'**-----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** Element-based surfaces \n');
fprintf(fid,'**
');
%
fprintf(fid,'*ELSET, ELSET=CAVITY1_B_e, GENERATE \n');
fprintf(fid,'%4d, %4d, %4d \n', 2000+n1-n1/d/2+1, 2000+n1, 1);
fprintf(fid,'%4d, %4d, %4d \n', 2001, 2000+n1/d/2, 1);
fprintf(fid,'*ELSET, ELSET=CAVITY1_F_e, GENERATE \n');
fprintf(fid,'%4d, %4d, %4d \n', 4401, 4400+2*n1/d, 1);
%
fprintf(fid,'*SURFACE, TYPE=ELEMENTS, NAME=CAVITY1_surv \n');
fprintf(fid,'CAVITY1_B_e, SNEG \n');
fprintf(fid,'%4d, SPOS \n', 3000+d);
fprintf(fid,'CAVITY1_F_e, SPOS \n');
fprintf(fid,'%4d, SNEG \n', 3000+d);
%
for i=2:d
    fprintf(fid,'*ELSET, ELSET=CAVITY%d_B_e, GENERATE \n', i );
    fprintf(fid,'%4d, %4d, %4d \n', 2001+n1/d/2+(i-2)*n1/d, 2000+3*n1/d/2+(i-2)*n1/d, 1);
    fprintf(fid,'*ELSET, ELSET=CAVITY%d_F_e, GENERATE \n', i );
    fprintf(fid,'%4d, %4d, %4d \n', 4001+n1/d/2+(i-2)*n1/d, 4000+3*n1/d/2+(i-2)*n1/d, 1);
%
    fprintf(fid,'*SURFACE, TYPE=ELEMENTS, NAME=CAVITY%d_surv \n', i);
    fprintf(fid,'CAVITY%d_B_e, SNEG \n', i);
    fprintf(fid,'%4d, SPOS \n', 3000+i-1 );
    fprintf(fid,'CAVITY%d_F_e, SPOS \n', i);
    fprintf(fid,'%4d, SNEG \n', 3000+i );
end
%

** Element properties

*BEAM SECTION, ELSET=TOWER_WALL_e, SECTION=RECT, MATERIAL=STEEL
0.05, 0.05
*BEAM SECTION, ELSET=APS_WALL_e, SECTION=RECT, MATERIAL=RUBBER_1
0.04, 0.01
*BEAM SECTION, ELSET=PARTITIONS_e, SECTION=RECT, MATERIAL=RUBBER_2
0.04, 0.04
*BEAM SECTION, ELSET=STIFFENERS_e, SECTION=RECT, MATERIAL=STEEL
0.1, 0.1
*BEAM SECTION, ELSET=SPRING_DASHPOT_e, SECTION=RECT, MATERIAL=SDM
0.1, 0.1

**

**

*MASS, ELSET=MASS2_e
%m2

**

** Materials

*MATERIAL, NAME=STEEL
*Density
7850.
*Elastic
200.E9, 0.3
*DAMPING, ALPHA= %3.2f, BETA= %3.2f

*MATERIAL, NAME=RUBBER_1
*Density
2000.
*Elastic
%4g, 0.49932
*DAMPING, ALPHA= %3.2g, BETA= %3.2g
fprintf(fid,'*MATERIAL, NAME=RUBBER_2 
');
fprintf(fid,'*Density 
');
fprintf(fid,'2000. 
');
fprintf(fid,'*Elastic 
');
fprintf(fid,'%4g, 0.49932 
', E_rubber_2);
fprintf(fid,'*DAMPING, ALPHA= %3.2g, BETA= %3.2g 
', alpha, beta);
%
fprintf(fid,'*MATERIAL, NAME=SDM 
');
fprintf(fid,'*Density 
');
fprintf(fid,'200. 
');
fprintf(fid,'*Elastic 
');
fprintf(fid,'%4g, 0.3 
', E_sdm);
fprintf(fid,'*DAMPING, ALPHA= %3.2g, BETA= %3.2g 
', alpha_sdm, beta_sdm);
fprintf(fid,'**
');
fprintf(fid,'**
');
fprintf(fid,'**----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** Definition of the surface based cavities  
');
fprintf(fid,'**
');
fprintf(fid,'*FLUID BEHAVIOR, NAME=Air 
');
fprintf(fid,'*CAPACITY, TYPE=POLYNOMIAL 
');
fprintf(fid,' 28.110, 1.967E-1 
');
fprintf(fid,' 28.110, 1.967E-3, 4.802E-6, -1.966E-9, 0.0 
');
fprintf(fid,'*MOLECULAR WEIGHT 
');
fprintf(fid,' 0.0289 
');
%
for i=1:d
    if ad==0
        fprintf(fid,'*FLUID CAVITY, NAME=CAVITY%d, SURFACE=CAVITY%d_surf, REF NODE=%d, AMBIENT=%4.2f, BEHAVIOR=Air \n', i, i, 100+i, p_a);
    else
        fprintf(fid,'*FLUID CAVITY, NAME=CAVITY%d, SURFACE=CAVITY%d_surf, REF NODE=%d, AMBIENT=%4.2f, BEHAVIOR=Air, ADIABATIC \n', i, i, 100+i, p_a);
    end
end
fprintf(fid,' 0.04 \n');
end
%
fprintf(fid,'*FLUID CAVITY, NAME=CAVITY301, ADDED VOLUME=10, REF NODE=301, AMBIENT=%4.2f, BEHAVIOR=Air \n', p_a );
else
fprintf(fid,'*FLUID CAVITY, NAME=CAVITY301, ADDED VOLUME=10, REF NODE=301, AMBIENT=%4.2f, BEHAVIOR=Air, ADIABATIC \n', p_a );
end
fprintf(fid,' 0.04 \n');
%
fprintf(fid,'*FLUID EXCHANGE, NAME=FLOW_OUT, PROPERTY=FLOW_OUT_PROP, EFFECTIVE AREA=1 \n');
fprintf(fid,'101 \n');
fprintf(fid,'*FLUID EXCHANGE, NAME=FLOW_IN, PROPERTY=FLOW_IN_PROP, EFFECTIVE AREA=1 \n');
fprintf(fid,'101, 301 \n');
fprintf(fid,'*FLUID EXCHANGE, NAME=FLOW_102, PROPERTY=FLOW_102_PROP, EFFECTIVE AREA=1 \n');
fprintf(fid,'101, 102 \n');
fprintf(fid,'*FLUID EXCHANGE, NAME=FLOW_%d, PROPERTY=FLOW_%d_PROP, EFFECTIVE AREA=1 \n', 100+d, 100+d);
fprintf(fid,'101, %d \n', 100+d);
%
fprintf(fid,'*FLUID EXCHANGE PROPERTY, NAME=FLOW_OUT_PROP, TYPE=BULK VISCOSITY \n');
fprintf(fid,' %4.2f \n', Cv1);
fprintf(fid,'*FLUID EXCHANGE PROPERTY, NAME=FLOW_IN_PROP, TYPE=BULK VISCOSITY \n');
fprintf(fid,' %4.2f \n', Cv2);
fprintf(fid,'*FLUID EXCHANGE PROPERTY, NAME=FLOW_102_PROP, TYPE=BULK VISCOSITY \n');
fprintf(fid,' %4.2f \n', Cv3);
fprintf(fid,'*FLUID EXCHANGE PROPERTY, NAME=FLOW_%d_PROP, TYPE=BULK VISCOSITY \n', 100+d);
fprintf(fid,' %4.2f \n', Cv4);
%
fprintf(fid,'**----------------------------------------------------------------------------
');
fprintf(fid,'**
');
fprintf(fid,'** Contact
');
fprintf(fid,'**
');
fprintf(fid,'*SURFACE, TYPE=SEGMENTS, NAME=SHIP_ms 
');
fprintf(fid,'START,  %3.3f, %3.3f   
',dist, -length/2 );
fprintf(fid,'LINE ,  %3.3f, %3.3f   
',dist, length/2 );
fprintf(fid, '*RIGID BODY, ANALYTICAL SURFACE=SHIP_ms, PIN NSET=PIN_n, REF NODE=999 \n');
fprintf(fid, '*ELEMENT, TYPE=mass, ELSET=MASS_e
');
fprintf(fid, '3, 777 \n');
fprintf(fid, '*MASS, ELSET=MASS_e \n');
fprintf(fid, '%4d \n', m);
fprintf(fid, '*SURFACE, TYPE=ELEMENTS, NAME=APS_out_ss \n');
fprintf(fid, 'APS_WALL_e, SNEG \n');
fprintf(fid, '*SURFACE, TYPE=ELEMENTS, NAME=APS_in_ss \n');
fprintf(fid, 'APS_WALL_e, SPOS \n');
fprintf(fid, '*SURFACE, TYPE=ELEMENTS, NAME=TOWER_ms \n');
fprintf(fid, 'TOWER_WALL_e, SNEG \n');

**-----------------------------------------------------------------------------
**
** Initial conditions
**
 *
fprintf(fid, '*INITIAL CONDITIONS, TYPE=TEMPERATURE  
');
for i=1:d
fprintf(fid, '%4d, %4.2f  
', 100+i, theta_0);
end
fprintf(fid, '*INITIAL CONDITIONS, TYPE=FLUID PRESSURE  
');
for i=2:d
fprintf(fid, '%d, %6.0f  
', 100+i, p2_0);
end
fprintf(fid, '*INITIAL CONDITIONS, TYPE=FLUID PRESSURE  
');
fprintf(fid, '301, %6.0f  
', p_in);
fprintf(fid, '*INITIAL CONDITIONS, TYPE=VELOCITY 
');
fprintf(fid, '999, 1, %4d  
', V);

fprintf(fid, '*AMPLITUDE, NAME=PRESS_CHANGE, DEFINITION=TABULAR, SMOOTH=0 \n');
fprintf(fid, '0,0, %4.2g,1 
', t1);
fprintf(fid, '0,0, %4.2g,0.25, %4.2g,0.5, %4.2g,0.75, %4.2g,1  
', 1*t1/4, 2*t1/4, 3*t1/4, t1);
fprintf(fid, '*AMPLITUDE, NAME=FLOW_CHANGE, DEFINITION=TABULAR \n');  % the flow is constant along the step time
fprintf(fid,'0, 1, 0.2, 1, 1.2, 1 \n');
%
fprintf(fid, '**-----------------------------------------------------------------------------
');
fprintf(fid, '**
');
fprintf(fid, '** History data
');
fprintf(fid, '**
');
%
fprintf(fid, '*STEP \n');
fprintf(fid, 'Explicit dynamics \n');
fprintf(fid, '*DYNAMIC, EXPLICIT \n');
fprintf(fid, ',%6f \n', t1 );
%
fprintf(fid, 'FLUID EXCHANGE INTERACTION \n');           % w pierwszym kroku analizy przepływ jest nieaktywny
fprintf(fid, 'FLOW_OUT, FLOW_IN, FLOW_102, FLOW_%d \n', 100+d);
%
fprintf(fid, '*CONTACT PAIR, INTERACTION=INT_1 \n');
fprintf(fid, 'APS_out_ss, SHIP_ms   \n');
fprintf(fid, '*CONTACT PAIR, INTERACTION=INT_2 \n');
fprintf(fid, 'APS_in_ss, TOWER_ms \n');
%
fprintf(fid, '*SURFACE INTERACTION, NAME=INT_1 \n');
fprintf(fid, '1.0  \n');
fprintf(fid, '*SURFACE INTERACTION, NAME=INT_2 \n');
fprintf(fid, '1.0  \n');
%
fprintf(fid, '*BOUNDARY \n');
fprintf(fid, '2, ENCASTRE\n');
fprintf(fid, '1, 3, 6 \n');
fprintf(fid, '999, 4, 6 \n');
%
fprintf(fid, '*BOUNDARY, AMPLITUDE=PRESS_CHANGE \n');
fprintf(fid, '101, 8, 8, %6.0f \n', p1_0);     % BC in the front cavity
fprintf(fid, '%d, 8, 8, %6.0f \n', 100+2, p2_0);
fprintf(fid, '%d, 8, 8, %6.0f \n', 100+d, p2_0);
for i=3:d-1
    fprintf(fid, '%d, 8, 8, %6.0f \n', 100+i, p3_0);
end
%fprintf(fid, '*BOUNDARY \n');
%fprintf(fid,'301, 8, 8, %4.2g \n', p_in); % BC at reference node 301
%
fprintf(fid,'**-----------------------------------------------------------------------------\n');
fprintf(fid,'**\n');
fprintf(fid,'** Output\n');
fprintf(fid,'**\n');
fprintf(fid,'*OUTPUT,FIELD, TIME INTERVAL=0.01 \n');
fprintf(fid,'*NODE OUTPUT \n');
fprintf(fid,'U \n');
fprintf(fid,'*NODE OUTPUT, NSET=REFERENCE_n \n');
fprintf(fid,'PCAV \n');
fprintf(fid,'CVOL \n');
fprintf(fid,'*ELEMENT OUTPUT \n');
fprintf(fid,'1, 3, 5 \n');
fprintf(fid,'S, SF \n');
%
fprintf(fid,'*OUTPUT, HISTORY, FREQUENCY=2 \n');
fprintf(fid,'*NODE OUTPUT, NSET=CENTER_n \n');
fprintf(fid,'U,a \n');
fprintf(fid,'*NODE OUTPUT, NSET=MASS1_n \n');
fprintf(fid,'U,a \n');
fprintf(fid,'*NODE OUTPUT, NSET=SHIP_n \n');
fprintf(fid,'U,a \n');
fprintf(fid,'*NODE OUTPUT, NSET=SUPPORT_n \n');
fprintf(fid,'RF, \n');
fprintf(fid,'*NODE OUTPUT, NSET=REFERENCE_n \n');
fprintf(fid,'PCAV \n');
fprintf(fid,'CVOL \n');
fprintf(fid,'CTEMP \n');
fprintf(fid,'CMASS \n');
%
fprintf(fid,'*ENERGY OUTPUT \n');
fprintf(fid,'ALLKE, ALLSE, ALLVD, ALLWK, ETOTAL \n');
%
fprintf(fid,'*ELEMENT OUTPUT, ELSET=HST_e \n');
fprintf(fid,'S11 \n');
fprintf(fid,'*ELEMENT OUTPUT, ELSET=HSR_e \n');
fprintf(fid,'S11 \n');
%
fprintf(fid,'*MONITOR, DOF=1, NODE=CENTER_n \n');
fprintf(fid,'*END STEP\n');
%
fprintf(fid,'**---------------------------------------------------------------\n');
fprintf(fid,'**---------------------------------------------------------------\n');
fprintf(fid,'** NEW STEP FOR REMOVING PRESS BC
**---------------------------------------------------------------\n');
fprintf(fid,'*STEP \n');
fprintf(fid,'Explicit dynamics \n');
fprintf(fid,'*DYNAMIC, EXPLICIT \n');
fprintf(fid,',%6f \n', t2 );
fprintf(fid,'*BOUNDARY, OP=NEW \n');
fprintf(fid,'2, ENCASTRE\n');
fprintf(fid,'1, 3, 6 \n');
fprintf(fid,'%d, 2, 2 \n', 1000*r1+n1/2);
fprintf(fid,'999, 4, 6 \n');
fprintf(fid,'**---------------------------------------------------------------\n');
fprintf(fid,'**
** Output
**
**
fprintf(fid,'*OUTPUT,FIELD, TIME INTERVAL=0.01 \n');
fprintf(fid,'*NODE OUTPUT \n');
fprintf(fid,'U \n');
fprintf(fid,'*NODE OUTPUT, NSET=REFERENCE_n \n');
fprintf(fid,'PCAV \n');
fprintf(fid,'CVOL \n');
fprintf(fid,'*ELEMENT OUTPUT \n');
fprintf(fid,'1, 3, 5 \n');
fprintf(fid,'S, SF \n');
fprintf(fid,'**---------------------------------------------------------------\n');
fprintf(fid,'*OUTPUT, HISTORY, FREQUENCY=2 \n');
fprintf(fid,'*NODE OUTPUT, NSET=CENTER_n \n');
fprintf(fid,'U,a \n');
fprintf(fid,'*NODE OUTPUT, NSET=MASS1_n \n');
fprintf(fid,'U,a \n');
fprintf(fid,'*NODE OUTPUT, NSET=SHIP_n \n');
fprintf(fid,'U,a \n');
fprintf(fid,'*NODE OUTPUT, NSET=SUPPORT_n \n');
fprintf(fid,'RF, \n');
fprintf(fid,'*NODE OUTPUT, NSET=REFERENCE_n \n');
fprintf(fid,'PCAV \n');
fprintf(fid,'CVOL \n');
fprintf(fid,'CTEMP \n');
fprintf(fid,'CMASS \n');
%
fprintf(fid,'*ENERGY OUTPUT \n');
fprintf(fid,'ALLKE, ALLSE, ALLVD, ALLWK, ETOTAL \n');
%
fprintf(fid,'*ELEMENT OUTPUT, ELSET=HST_e \n');
fprintf(fid,'S11 \n');
fprintf(fid,'*ELEMENT OUTPUT, ELSET=HSR_e \n');
fprintf(fid,'S11 \n');
%
fprintf(fid,'*MONITOR, DOF=1, NODE=CENTER_n \n');
fprintf(fid,'*END STEP
');
fclose(fid);
Appendix C: FORTRAN subroutine for adiabatic change of gas

SUBROUTINE URDFIL(LSTOP, LOVRWRT, KSTEP, KINC, DTIME, TIME)

INCLUDE 'ABA_PARAM.INC'

DIMENSION ARRAY(513), JRARRAY(NPRECD, 513), TIME(2)
EQUIVALENCE (ARRAY(1), JRARRAY(1, 1))
PARAMETER(nodenumber=101)

open(108, file='c:\temp\all_out.dat')
open(109, file='c:\temp\urdfil_out.dat')

C FIND CURRENT INCREMENT

CALL POSFIL(KSTEP, KINC, ARRAY, JRCD)
DO K1=1, 999999
  CALL DBFILE(0, ARRAY, JRCD)
  IF (JRCD .NE. 0) GO TO 200
  KEY=JRARRAY(1, 2)
  node=JRARRAY(1, 3)
  C   136 CONTAINS VALUES FOR PCAV AT NODE 101

  IF (KEY.EQ.136) THEN
    IF (node.eq.nodenumber) then
      pcav_new = array(4)
      write(109,904) pcav_new
      write(108,*) 'I am in URDFIL'
      write(108,*) kinc, TIME(1), pcav_new
      write(108,*) ' ----------------------------------  '
    end if
  END IF

  IF (KEY.EQ.137) THEN
    IF (node.eq.nodenumber) then
      cvol_new = array(4)
      write(109,904) pcav_new, cvol_new
      write(108,*) 'I am in URDFIL'
      write(108,903) kinc, array(1:3), array(4:6), pcav_new, cvol_new
  END IF
! C2

write(108,*) kinc, TIME(1), pcav_new, cvol_new
write(108,*) ' ----------------------------------  
END IF

END DO

200 CONTINUE

903 format(4i7,4G12.4)
904 format(6G12.4)

CLOSE(109)

RETURN

END

SUBROUTINE UTEMP(TEMP,NSECPT,KSTEP,KINC,TIME,NODE,COORDS)
C
INCLUDE 'ABA_PARAM.INC'
C
DIMENSION TEMP(NSECPT), TIME(2), COORDS(3)
C
REAL pcav_old, cvol_old, temp_old, pcav_new, temp_new, cv, obl, m
REAL pcav_0, temp_0

open(108,file='c:\temp\all_out.dat')
open(109,file='c:\temp\urdfil_out.dat')
open(110,file='c:\temp\utemp_out.dat')

READ(109, *, END=1000) pcav_new, cvol_new
1000 CONTINUE
READ(110, *, END=2000) pcav_old, cvol_old, temp_old
2000 CONTINUE
CLOSE(110)

cvol_0 = 0.098027
temp_0 = 1.117550
cv = (28.11-8.3413)/0.0289
m = ((50e3+101360)*cvol_0)/8.34134*0.0289/(temp_0+273.15)
obl = -(pcav_old+pcav_new+2*101360)/2*(cvol_new-cvol_old)/cv/m
 TEMP(1) = obl + temp_old

pcav_old = pcav_new
cvol_old = cvol_new
temp_old = TEMP(1)

write(108,*) 'I am in UFIELD'
write(108,*) KINC, TIME(1), obl, TEMP(1)

open(110,file='c:\temp\utemp_out.dat')
write(110,*) pcav_old, cvol_old, temp_old

903 format(4i7,4G12.4) 904 format(6G12.4)

CLOSE(109)
CLOSE(110)

RETURN
END

FORTRAN subroutine for adiabatic flow through orifice:

C SUBROUTINE URDFIL(LSTOP,LOVRWRT,KSTEP,KINC,DTIME,TIME)
C INCLUDE 'ABA_PARAM.INC'
C DIMENSION ARRAY(513),JRRAY(NPRECD,513),TIME(2)
EQUIVALENCE (ARRAY(1),JRRAY(1,1))
PARAMETER(nodenumber=101)
C open(108,file='c:\temp\all_out.dat')
open(109,file='c:\temp\urdfil_out.dat')
C C FIND CURRENT INCREMENT
C CALL POSFIL(KSTEP,KINC,ARRAY,JRCRD)
DO K1=1,999999
   CALL DBFILE(0,ARRAY,JRCD)
   IF (JRCD .NE. 0) GO TO 200
   KEY=JRRAY(1,2)
   node=JRRAY(1,3)
C 136 CONTAINS VALUES FOR PCAV AT NODE 101
C
   IF (KEY.EQ.136) THEN
      IF (node.eq.nodenum) then
         pcav=ARRAY(4)
         WRITE(109,904) pcav
         WRITE(108,*) 'I am in URDFIL'
         WRITE(108,903) kinc, array(1:3), array(4:6), pcav
         WRITE(108,*) kinc, TIME(1), pcav
         WRITE(108,*) ' ----------------------------------  '
      end if
   END IF
END DO

200 CONTINUE

903 FORMAT(4I7,4G12.4)
904 FORMAT(6G12.4)

CLOSE(109)
RETURN
END

SUBROUTINE
UFIELD(FIELD,KFIELD,NSECPT,KSTEP,KINC,TIME,NODE,COORDS,
   TEMP,DTEMP,NFIELD)
C
   INCLUDE 'ABA_PARAM.INC'
C
   DIMENSION FIELD(NSECPT,NFIELD),COORDS(3),TIME(2),TEMP(NSECPT),
   DTEMP(NSECPT)
C
   REAL C,a,b,R,theta,p_a,cp,kappa,p_e_total,p_c,p,p1,nawias,p2
C REAL C,a,b,R,theta,p_a,cp,kappa,p_e_total,p_c,p,p1,nawias,p2

C = 1e-5

a = 28.1100
b = 0.001967
R = 8.314
theta = 20
p_a = 100000

cp = a + b*(theta+273.15)
kappa = cp/(cp-R)

p_e_total = pcav + p_a
p_c = p_e_total *((2/(kappa+1))**(kappa/(kappa-1)))

IF (p_a < p_c) THEN
  p = p_c
ELSE
  p = p_a
END IF

p1 = sqrt(R/0.0289*(theta+273.15))
nawias = (p/p_e_total)**(2/(kappa)-2) - ((p/p_e_total)**((kappa+1)/kappa))**kappa
p2 = sqrt(2*kappa/nawias)

FIELD(1,1)= (pcav)*p1/(C*p_e_total*p2)

FIELD(1,1)= (pcav)*2e6

write(108,*) 'I am in UFIELD'
write(108,*) KINC, TIME(1), FIELD(1,1)
write(108,*) KINC, TIME(1), pcav, p_e_total, p_c
write(108,*) p, p1, nawias, p2, FIELD(1,1)
FORTRAN subroutine for dissipating impact energy:

```
SUBROUTINE URDFIL(LSTOP,LOVRWRT,KSTEP,KINC,DTIME,TIME)

INCLUDE 'ABA_PARAM.INC'

DIMENSION ARRAY(513),JRRAY(NPRECD,513),TIME(2)
EQUIVALENCE (ARRAY(1),JRRAY(1,1))
PARAMETER(nodenumber=999)

open(108,file='c:\temp\all_out.dat')
open(109,file='c:\temp\urdfil_out.dat')

C FIND CURRENT INCREMENT
C
CALL POSFIL(KSTEP,KINC,ARRAY,JRCD)
DO K1=1,999999
    CALL DBFILE(0,ARRAY,JRCD)
    IF (JRCD .NE. 0) GO TO 200
    KEY=JRRAY(1,2)
    node=JRRAY(1,3)
    C 136 CONTAINS VALUES FOR PCAV AT NODE 101
    IF (KEY.EQ.102) THEN
        if (node.eq.nodenumber) then
            vx=array(4)
            write(109,904) vx
            write(108,*) 'I am in URDFIL'
            write(108,*) kinc,TIME(1),vx
            write(108,903) kinc, array(1:3), array(4:6), vx
            write(108,903) kinc, TIME(1), vx
```

```
SUBROUTINE UFIELD(FIELD,KFIELD,NSECPT,KSTEP,KINC,TIME,NODE,COORDS,
1                  TEMP,DTEMP,NFIELD)
C
INCLUDE 'ABA_PARAM.INC'
C
DIMENSION FIELD(NSECPT,NFIELD),COORDS(3),TIME(2),TEMP(NSECPT),
1                  DTEMP(NSECPT)
C
REAL

open(108,file='c:\temp\all_out.dat')
onpen(109,file='c:\temp\urdfil_out.dat')

READ(109, *, END=1000) vx
1000 CONTINUE

REAL vx, valve_c, E_part

IF (vx < p_c) THEN
    valve_c = 1e10
    E_part = 150
ELSE
    valve_c = 2e5
END IF

END IF

END DO

200 CONTINUE

903 format(4i7,4G12.4)
904 format(6G12.4)

CLOSE(109)

RETURN

END
E_part = 1.5
END IF

FIELD(1,1) = valve_c

FIELD(1,2) = E_part

write(108,*) 'I am in UFIELD'
C write(108,*) KINC, TIME(1), FIELD(1,1)
write(108,*) KINC, TIME(1), vx, FIELD(1,1), FIELD(1,2)

903 format(4i7,4G12.4)
904 format(6G12.4)
C format(F4.1, 2X, F4.1, 2X, F7.5, 2X, F10.8, 2X, F10.8, 2X,
C F9.1,2X, F9.1, 2X, F9.1, 2X, F9.1 )

CLOSE(109)

RETURN
END

FORTRAN subroutine for mitigation of ship accelerations:

C SUBROUTINE URDFIL(LSTOP,LOVRWRT,KSTEP,KINC,DTIME,TIME)
C INCLUDE 'ABA_PARAM.INC'
C DIMENSION ARRAY(513),JRRAY(NPRECD,513),TIME(2)
EQUIVALENCE (ARRAY(1),JRRAY(1,1))
PARAMETER(nodenumber=999)
C
open(108,file='c:\temp\all_out.dat')
open(109,file='c:\temp\urdfil_out.dat')
C
C FIND CURRENT INCREMENT
C CALL POSFIL(KSTEP,KINC,ARRAY,JRCD)
DO K1 = 1,9999999
    CALL DBFILE(0,ARRAY,JRCD)
    IF (JRCD .NE. 0) GO TO 200
KEY=JRRAY(1,2)
    node=JRRAY(1,3)

C

C 136 CONTAINS VALUES FOR PCAV AT NODE 101
C

IF (KEY.EQ.103) THEN
    if (node.eq.nodenumber) then
        ax = array(4)
        write(109,904) ax
        write(108,*) 'I am in URDFIL'
    write(108,903) kinc, array(1:3), array(4:6), ux, p_old
    write(108,*) kinc, TIME(1), ax
    write(108,*) ' ----------------------------------  '  
    end if
END IF

END DO

200 CONTINUE

CLOSE(109)

C

903 format(4i7,4G12.4)
904 format(6G12.4)

RETURN
END

SUBROUTINE DISP(U,KSTEP,KINC,TIME,NODE,NOEL,JDOF,COORDS)

INCLUDE 'ABA_PARAM.INC'

DIMENSION U(3),TIME(2),COORDS(3)

REAL k, u_opt, du, dp, p_old

OPEN(108, FILE = 'c:\temp\all_out.dat')
OPEN(109, FILE = 'c:\temp\urdfil_out.dat')
OPEN(110, FILE = 'c:\temp\disp_out.dat')

READ(109, *, END=1000) ax

1000 CONTINUE
READ(110, *, END=2000) p_old

2000 CONTINUE
CLOSE(110)

dp = 1000*(17-ax)/0.08

IF (ax < 1) THEN
    U(1) = p_old
ELSE IF (TIME(1) > 0.6) THEN
    U(1) = p_old
ELSE IF (ax > 1) THEN
    U(1) = p_old + 0.001*dp
END IF

write(108,*) 'I am in DISP'
write(108,*) KINC, TIME(1), U(1)
C write(108,*) KINC, TIME(1), du, p_old, dp, U(1)

open(110,file='c:\temp\disp_out.dat')
write(110,*) U(1)

903  format(4i7,4G12.4)
904  format(6G12.4)

CLOSE(109)
CLOSE(110)

RETURN

END
VTT Working papers

43  Tsupari, Eemeli, Tormonen, Kauko, Monni, Suvi, Vahlman, Tuula, Kolsi, Aimo & Linna, Veli. Dityyppioksidin (N₂O) ja metaanin (CH₄) päästökertoimia Suomen voimalaitoksille, lämpökeskuksille ja pienpoltoille. 2006. 94 s. + liitt. 7 s.
44  Saarinen, Jani, Rilla, Nina, Loikkanen, Torsti, Oksanen, Juha & Alasaarela, Jaakko. Innovaatioympäristö tänään ja huomenna. 2006. 32 s.
50  Törnvist, Jouko & Talja, Asko. Suositus liikenneturin arvioimiseksi maankäytön suunnittelussa. 2006. 46 s. + liitt. 33 s.
53  Pasonen, Markku & Hakkarainen, Toni. Kaukolämpölinjojen elinkäja ja NDT. 2006. 27 s.
57  Kulmala, Risto. Tiehallinto ja liikenteen tietopalvelut. Selvitysmiehen muistio. 2006. 29 s. + liitt. 3 s.